
IT'S NOT MY FAULT—THE GAS PEDAL'S STUCK...
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WALL ST.
ADAM ZYGLIS ©2010 CAGLECARTOONS.COM THEBUFFALONEWS

Financial Resources

Does the major clearinghouse currently have adequate financial resources to deal with a major multiple bank default?[1]

Note: [1]Based on 62 respondents in 2014. [2]Based on 72 respondents in 2014.
Source: Greenwich Associates 2014 Systemic Risk and the Impacts of Central Clearing Study

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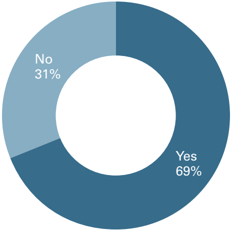

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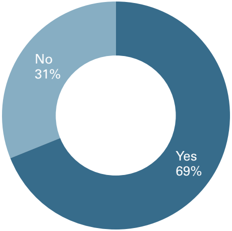

What can be done to limit the likelihood that a clearinghouse will fail[2]

Access to central bank liquidity

Better capitalized/clearinghouses having "more skin in the game"

Higher default fund requirement

Higher quality collateral requirements

More stringent membership requirements

Higher margin requirements

More members

Risk models with longer look back period

Other

More stringent product selection criteria

Nothing beyond current practices

Fewer members

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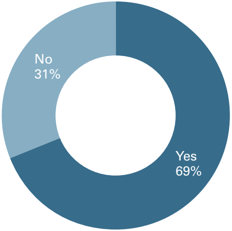

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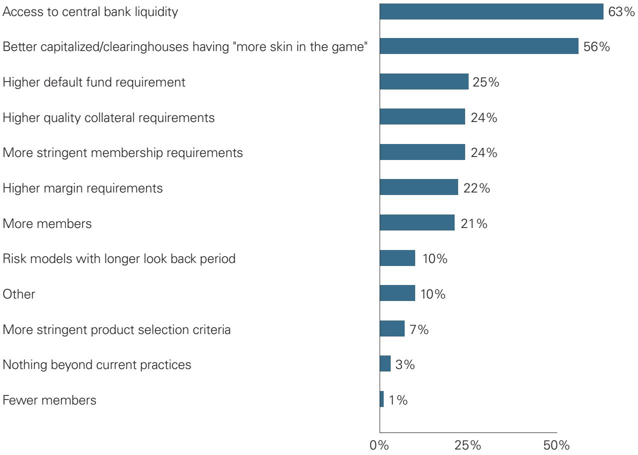

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Crowded Risk as a Systemic Concern for Central Clearing Counterparties

Albert J. Menkveld

VU University Amsterdam and Tinbergen Institute

October 20, 2015

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1. Bernanke (2011) emphasized financial stability strongly depends on resiliency of CCP.
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   However, the more prominent role of CCPs will also introduce new systemic risks. Mandatory clearing will turn CCPs into systemic nodes in the financial system, with unknown, but possibly far-reaching, consequences.

Exhibit 1:
CPSS-IOSCO Technical Committee
Recommendations for Central Counterparties (CCPs)

1. Legal risk

A CCP should have a well founded, transparent and enforceable legal framework for each aspect of its activities in all relevant jurisdictions.

2. Participation requirements

A CCP should require participants to have sufficient financial resources and robust operational capacity to meet obligations arising from participation in the CCP. A CCP should have procedures in place to monitor that participation requirements are met on an ongoing basis. A CCP's participation requirements should be objective, publicly disclosed, and permit fair and open access.

3. Measurement and management of credit exposures

A CCP should measure its credit exposures to its participants at least once a day. Through margin requirements, other risk control mechanisms or a combination of both, a CCP should limit its exposures to potential losses from defaults by its participants in normal market conditions so that the operations of the CCP would not be disrupted and non-defaulting participants would not be exposed to losses that they cannot anticipate or control.

4. Margin requirements

If a CCP relies on margin requirements to limit its credit exposures to participants, those requirements should be sufficient to cover potential exposures in normal market conditions. The models and parameters used in setting margin requirements should be risk-based and reviewed regularly.

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3. For example, 54 exchanges and clearing houses use SPAN developed by Chicago Mercantile Exchange (CME).

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Objective

1. Do crowded trades constitute a hidden risk to a CCP not accounted for by member-by-member margins? Yes!
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3. And, what is the appropriate way to calculate of CCP (tail) risk? Once established, is there a natural way to allocate it across members (according to the “polluter pays” principle)? Yes!

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   3.2 A new tail risk calculation, Margin(A), to appropriately account crowded-trade risk.

Literature

1. **CCP vs. OTC**
   Duffie and Zhu (2011), Koeppl, Monnet, and Temzelides (2012), Menkveld, Pagnotta, and Zoican (2013).
2. **Counterparty risk monitoring**
   Biais, Heider, and Hoerova (2011), Acharya and Bisin (2011), Koeppl (2013).
3. **Systemic risk in trades**
   Basak and Shapiro (2001), Acharya (2009), Farhi and Tirole (2012).
4. **CCP risk management**
   Cruz Lopez et al. (2014), Hedegaard (2012), Jones and Pérignon (2013), Menkveld (2013).
5. **Crowded trades**
   Khandani and Lo (2007), Khandani and Lo (2011), Pojarliev and Levich (2011).
6. **Systemic risk allocation**
   Brunnermeier and Cheridito (2014), . . .

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4. Let $X_j = n_j' R$ be the P&L on member j's trade portfolio, then

$$X \sim \mathrm{N}(0, \Sigma), \quad \Sigma = N' \Omega N, \quad N = [n_1, \cdots, n_J].$$

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$$X \sim \mathrm{N}(0, \Sigma), \quad \Sigma = N' \Omega N, \quad N = [n_1, \cdots, n_J]\,.$$

5. CCP aggregate exposure to trade portfolios of all members is defined as

$$A = \sum_j E_j \quad \text{with } E_j = -\min\left(X_j, 0\right).$$

Measure

1. Duffie and Zhu (2011) calculate aggregate exposure mean to derive their main result.

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1. Duffie and Zhu (2011) calculate aggregate exposure mean to derive their main result.
2. Can its standard deviation also be derived analytically?

Absolute Moments in 2-dimensional Normal Distribution

By Seiji NABEYA

Let x and y be distributed according to the following 2-dimensional normal distribution,

$$\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}\exp\left\{-\frac{1}{2(1-\rho^2)}\left(\frac{x^2}{\sigma_1^2}-\frac{xy}{\sigma_1\sigma_2}+\frac{y^2}{\sigma_2^2}\right)\right\}dx\,dy.$$

It is our purpose to express absolute moments in terms of elementary functions. Putting $E(|x^m y^n|)=(m,n)$ for simplicity, we have

$$(m,n)=\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|x^m y^n|\exp\left\{-\frac{1}{2(1-\rho^2)}\right.$$

$$\left.\times\left(\frac{x^2}{\sigma_1^2}-2\rho\frac{xy}{\sigma_1\sigma_2}+\frac{y^2}{\sigma_2^2}\right)\right\}dx\,dy$$

$$=\frac{2^{\frac{m+n}{2}}\sigma_1^m\sigma_2^n}{\pi}(1-\rho^2)^{\frac{m+n+1}{2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|x^m y^n|\exp(-x^2+2\rho xy-y^2)\,dx\,dy$$

$$=\frac{2^{\frac{m+n}{2}}\sigma_1^m\sigma_2^n}{\pi}(1-\rho^2)^{\frac{m+n+1}{2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|x^m y^n|e^{-x^2-y^2}\sum_{k=0}^{\infty}\frac{(2\rho xy)^k}{k\,!}\,dx\,dy$$

$$=\frac{2^{\frac{m+n}{2}}\sigma_1^m\sigma_2^n}{\pi}(1-\rho^2)^{\frac{m+n+1}{2}}\sum_{k=0}^{\infty}\frac{\Gamma\left(\frac{m+1}{2}+k\right)\Gamma\left(\frac{n+1}{2}+k\right)}{(2k)\,!}(2\rho)^{2k}$$

$$=\frac{2^{\frac{m+n}{2}}\sigma_1^m\sigma_2^n}{\pi}\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+1}{2}\right)(1-\rho^2)^{\frac{m+n+1}{2}}$$

$$\times F\left(\frac{m+1}{2},\frac{n+1}{2};\frac{1}{2};\rho^2\right)$$

$$=\frac{2^{\frac{m+n}{2}}\sigma_1^m\sigma_2^n}{\pi}\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+1}{2}\right)F\left(-\frac{m}{2},-\frac{n}{2};\frac{1}{2};\rho^2\right).$$

Here

$$F(\alpha,\beta;\gamma;z)=1+\frac{\alpha\beta}{1!\,\gamma}z+\frac{\alpha(\alpha+1)\beta(\beta+1)}{2!\,\gamma(\gamma+1)}z^2+\cdots\cdots$$

is the hypergeometric function, which reduces to the polynomial of z if α or β is a non-positive integer and γ is positive. Thus, when at least one of the integers m, n is an even number, (m,n) reduces to the polynomial of ρ^2 multiplied by $\sigma_1^m\sigma_2^n$.

The case where both m and n are odd may be treated as follows. Put

$$x = \sqrt{2(1-\rho^2)}\,\sigma_1 r\cos\theta, \qquad y = \sqrt{2(1-\rho^2)}\,\sigma_2 r\sin\theta.$$

When $m-n=2q$, where q is a non-negative integer, we have then

$$(m,n) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|x^m y^n|\exp\left\{-\frac{1}{2(1-\rho^2)}\right.$$

$$\left.\times\left(\frac{x^2}{\sigma_1^2}-2\rho\frac{xy}{\sigma_1\sigma_2}+\frac{y^2}{\sigma_2^2}\right)\right\}dx\,dy$$

$$=\frac{2^{\frac{m+n}{2}}\sigma_1^m\sigma_2^n}{\pi}(1-\rho^2)^{\frac{m+n+1}{2}}\int_0^{2\pi}\int_0^{\infty}r^{m+n+1}|\cos^m\theta\sin^n\theta|$$

$$\times\exp\left\{-r^2(1-2\rho\cos\theta\sin\theta)\right\}dr\,d\theta$$

$$=\frac{2^{\frac{m+n-2}{2}}\sigma_1^m\sigma_2^n}{\pi}\Gamma\left(\frac{m+n}{2}+1\right)(1-\rho^2)^{\frac{m+n+1}{2}}$$

$$\times\int_0^{2\pi}\frac{|\cos^m\theta\sin^n\theta|}{(1-2\rho\cos\theta\sin\theta)^{\frac{m+n+2}{2}}}d\theta$$

$$=\frac{2^{\frac{m+n}{2}}\sigma_1^m\sigma_2^n}{\pi}\Gamma\left(\frac{m+n}{2}+1\right)(1-\rho^2)^{\frac{m+n+1}{2}}$$

$$\times\int_0^{\frac{\pi}{2}}\left\{\frac{\cos^m\theta\sin^n\theta}{(1-2\rho\cos\theta\sin\theta)^{\frac{m+n+2}{2}}}+\frac{\cos^m\theta\sin^n\theta}{(1+2\rho\cos\theta\sin\theta)^{\frac{m+n+2}{2}}}\right\}d\theta$$

$$=\frac{2^{\frac{m-n}{2}}\sigma_1^m\sigma_2^n}{\pi}\Gamma\left(\frac{m-n}{2}+1\right)(1-\rho^2)^{\frac{m+n+1}{2}}$$

$$\times\frac{d^n}{d\rho^n}\int_0^{\frac{\pi}{2}}\left\{\frac{\cos^{2q}\theta}{(1-2\rho\cos\theta\sin\theta)^{q+1}}-\frac{\cos^{2q}\theta}{(1+2\rho\cos\theta\sin\theta)^{q+1}}\right\}d\theta.$$

As the last integral may be calculated in the elementary fashion, (m, n) may be evaluated.

In the following we shall give the obatined formulae for the cases $m \geqq n$. The formula of (m, n) for $m \leqq n$, is obtained by exchanging σ_1 and σ_2 in the formula (n, m).

$$(1,0)=\sqrt{\frac{2}{\pi}}\,\sigma_1,$$

$$(2,0)=\sigma_1^2,$$

$$(1,1)=\frac{2}{\pi}\left(\sqrt{1-\rho^2}+\rho\sin^{-1}\rho\right)\sigma_1\sigma_2,$$

$$(3,0) = 2\sqrt{\frac{2}{\pi}}\,\sigma_1^3,$$

$$(2,1) = \sqrt{\frac{2}{\pi}}\,(1+\rho^2)\,\sigma_1^2\sigma_2,$$

$$(4,0) = 3\sigma_1^4,$$

$$(3,1) = \frac{2}{\pi}\left\{\sqrt{1-\rho^2}\,(2+\rho^2) + 3\rho\sin^{-1}\rho\right\}\sigma_1^3\sigma_2.$$

$$(2,2) = (1+2\rho^2)\,\sigma_1^2\sigma_2^2,$$

$$(5,0) = 8\sqrt{\frac{2}{\pi}}\,\sigma_1^5,$$

$$(4,1) = \sqrt{\frac{2}{\pi}}\,(3+6\rho^2-\rho^4)\,\sigma_1^4\sigma_2,$$

$$(3,2) = 2\sqrt{\frac{2}{\pi}}\,(1+3\rho^2)\,\sigma_1^3\sigma_2^2,$$

$$(6,0) = 15\sigma_1^6,$$

$$(5,1) = \frac{2}{\pi}\left\{\sqrt{1-\rho^2}\,(8+9\rho^2-2\rho^4) + 15\rho\sin^{-1}\rho\right\}\sigma_1^5\sigma_2,$$

$$(4,2) = 3(1+4\rho^2)\sigma_1^4\sigma_2^2,$$

$$(3,3) = \frac{2}{\pi}\left\{\sqrt{1-\rho^2}\,(4+11\rho^2) + 3\rho(3+2\rho^2)\sin^{-1}\rho\right\}\sigma_1^3\sigma_2^3,$$

$$(7,0) = 48\sqrt{\frac{2}{\pi}}\,\sigma_1^7,$$

$$(6,1) = 3\sqrt{\frac{2}{\pi}}\,(5+15\rho^2-5\rho^4+\rho^6)\,\sigma_1^6\sigma_2,$$

$$(5,2) = 8\sqrt{\frac{2}{\pi}}\,(1+5\rho^2)\,\sigma_1^5\sigma_2^2,$$

$$(4,3) = 6\sqrt{\frac{2}{\pi}}\,(1+6\rho^2+\rho^4)\,\sigma_1^4\sigma_2^3,$$

$$(8,0) = 105\sigma_1^8,$$

$$(7,1) = \frac{2}{\pi}\left\{\sqrt{1-\rho^2}\,(48+87\rho^2-38\rho^4+8\rho^6) + 105\rho\sin^{-1}\rho\right\}\sigma_1^7\sigma_2,$$

$$(6,2) = 15(1+6\rho^2)\,\sigma_1^6\sigma_2^2,$$

$$(5,3) = \frac{2}{\pi}\left\{\sqrt{1-\rho^2}\,(16+83\rho^2+6\rho^4) + 15\rho(3+4\rho^2)\sin^{-1}\rho\right\}\sigma_1^5\sigma_2^3,$$

$$(4,4) = 3(3+24\rho^2+8\rho^4)\,\sigma_1^4\sigma_2^4,$$

$$(9,0) = 384\sqrt{\frac{2}{\pi}}\,\sigma_1^9,$$

$$(8,1) = 3\sqrt{\frac{2}{\pi}}\,(35 + 140\rho^2 - 70\rho^4 + 28\rho^6 - 5\rho^8)\,\sigma_1^8\sigma_2,$$

$$(7,2) = 48\sqrt{\frac{2}{\pi}}\,(1 + 7\rho^2)\,\sigma_1^7\sigma_2^2,$$

$$(6,3) = 6\sqrt{\frac{2}{\pi}}\,(5 + 45\rho^2 + 15\rho^4 - \rho^6)\,\sigma_1^6\sigma_2^3,$$

$$(5,4) = 24\sqrt{\frac{2}{\pi}}\,(1 + 10\rho^2 + 5\rho^4)\,\sigma_1^5\sigma_2^4,$$

$$(10,0) = 945\sigma_1^{10},$$

$$(9,1) = \frac{6}{\pi}\Big\{\sqrt{1-\rho^2}\,(128 + 325\rho^2 - 210\rho^4 + 88\rho^6 - 16\rho^8) + 315\rho\sin^{-1}\rho\Big\}\,\sigma_1^9\sigma_2,$$

$$(8,2) = 105(1 + 8\rho^2)\,\sigma_1^8\sigma_2^2,$$

$$(7,3) = \frac{2}{\pi}\Big\{\sqrt{1-\rho^2}\,(96 + 741\rho^2 + 120\rho^4 - 12\rho^6) + 315\rho(1 + 2\rho^2)\sin^{-1}\rho\Big\}\,\sigma_1^7\sigma_2^3,$$

$$(6,4) = 45(1 + 12\rho^2 + 8\rho^4)\sigma_1^6\sigma_2^4,$$

$$(5,5) = \frac{2}{\pi}\Big\{\sqrt{1-\rho^2}\,(64 + 607\rho^2 + 274\rho^4) + 15\rho(15 + 40\rho^2 + 8\rho^4)\sin^{-1}\rho\Big\}\sigma_1^5\sigma_2^5,$$

$$(11,0) = 3840\sqrt{\frac{2}{\pi}}\,\sigma_1^{11},$$

$$(10,1) = 15\sqrt{\frac{2}{\pi}}\,(63 + 315\rho^2 - 210\rho^4 + 126\rho^6 - 45\rho^8 + 7\rho^{10})\sigma_1^{10}\sigma_2,$$

$$(9,2) = 384\sqrt{\frac{2}{\pi}}\,(1 + 9\rho^2)\sigma_1^9\sigma_2^2,$$

$$(8,3) = 6\sqrt{\frac{2}{\pi}}\,(35 + 420\rho^2 + 210\rho^4 - 28\rho^6 + 3\rho^8)\sigma_1^8\sigma_2^3,$$

$$(7,4) = 48\sqrt{\frac{2}{\pi}}\,(3 + 42\rho^2 + 35\rho^4)\sigma_1^7\sigma_2^4,$$

$$(6,5) = 120\sqrt{\frac{2}{\pi}}\,(1 + 15\rho^2 + 15\rho^4 + \rho^6)\sigma_1^6\sigma_2^5,$$

$$(12,0) = 10395\sigma_1^{12},$$

$$(11,1) = \frac{6}{\pi}\left\{\sqrt{1-\rho^2}\,(1280 + 4215\rho^2 - 3590\rho^4 + 2248\rho^6 - 816\rho^8 + 128\rho^{10}) + 3465\rho\sin^{-1}\rho\right\}\sigma_1^{11}\sigma_2,$$

$$(10,2) = 945(1+10\rho^2)\sigma_1^{10}\sigma_2^2,$$

$$(9,3) = \frac{6}{\pi}\left\{\sqrt{1-\rho^2}\,(256 + 2639\rho^2 + 690\rho^4 - 136\rho^6 + 16\rho^8) + 315\rho(3+8\rho^2)\sin^{-1}\rho\right\}\sigma_1^9\sigma_2^3,$$

$$(8,4) = 315(1 + 16\rho^2 + 16\rho^4)\sigma_1^8\sigma_2^4,$$

$$(7,5) = \frac{6}{\pi}\left\{\sqrt{1-\rho^2}\,(128 + 1779\rho^2 + 1518\rho^4 + 40\rho^6) + 105\rho(5 + 20\rho^2 + 8\rho^4)\sin^{-1}\rho\right\}\sigma_1^7\sigma_2^5,$$

$$(6,6) = 45(5 + 90\rho^2 + 120\rho^4 + 16\rho^6)\sigma_1^6\sigma_2^6.$$

In another paper we shall treat the 3-dimensional case by a unified but more complicated method.

Institute of Statistical Mathematics.

Measure

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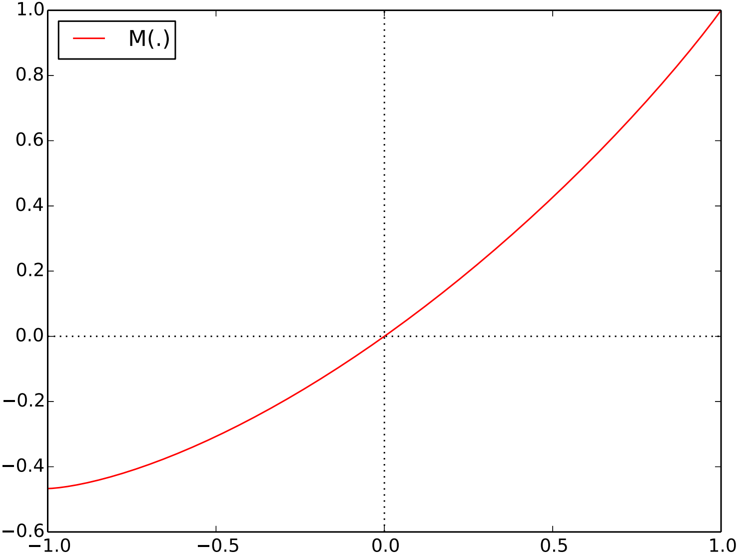
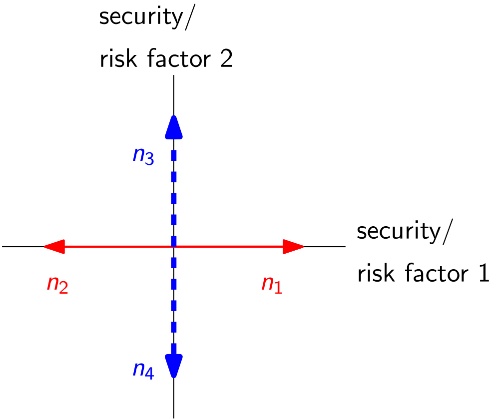
$$M(\rho) = \frac{\left[\frac{1}{2}\pi + \arcsin(\rho)\right]\rho + \sqrt{1-\rho^2} - 1}{\pi - 1}$$

Measure



Noncrowded trades



Simple example noncrowded trades

1.
$$N = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}, \ \Sigma = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

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$$E(E) = \sqrt{\frac{1}{2\pi}} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \; var(E) = \frac{1}{2\pi} \begin{pmatrix} \pi - 1 & -1 & 0 & 0 \\ -1 & \pi - 1 & 0 & 0 \\ 0 & 0 & \pi - 1 & -1 \\ 0 & 0 & -1 & \pi - 1 \end{pmatrix}$$

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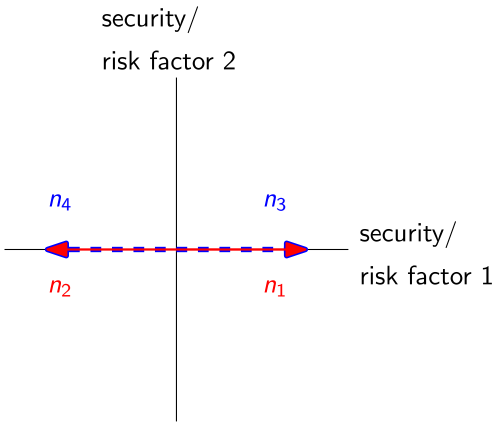
3.

$$E(A) = 4\sqrt{\frac{1}{2\pi}} \approx 1.60 \quad \text{and} \quad std(A) = 2\sqrt{\frac{\pi - 2}{2\pi}} \approx 0.85$$

Crowded trades



Simple example crowded trades

1.

$$N = \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \; \Sigma = N'\Omega N = \begin{pmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{pmatrix}$$

Simple example crowded trades

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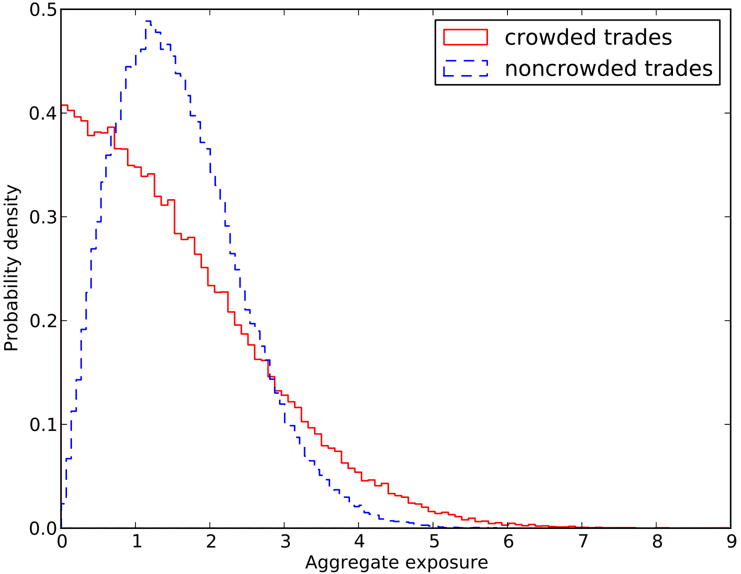
3.

$$E(A) = 4\sqrt{\frac{1}{2\pi}} \approx 1.60 \quad \text{and} \quad \text{std}(A) = 2\sqrt{\frac{\pi - 2}{\pi}} \approx 1.21$$

Histogram aggregate exposure for four members (N=4)



A crowded-trade risk thermometer?

Is there a natural "thermometer" for crowded-trade risk?

[1] A feasible approach to this NP hard problem is to convert it to a standard bin-packing problem which can be "solved" heuristically (see Appendix A of the slides).

A crowded-trade risk thermometer?

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Definition

CrowdIx for Σ is defined as

$$CrowdIx = std(A)/std(\tilde{A})$$

where $\tilde{A}$ is CCP aggregate exposure when all members' trades are re-allocated to a single risk factor to the maximum extent possible.[1]

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Lemma

$$CrowdIx \geq \sqrt{\frac{1}{\tilde{J}/2}} \qquad where\ \tilde{J} = 2\lfloor J/2 \rfloor J$$

[1] A feasible approach to this NP hard problem is to convert it to a standard bin-packing problem which can be “solved” heuristically (see Appendix A of the slides).

A crowded-trade risk thermometer?

1. CrowdIx in the simple example is
$$\begin{cases} \sqrt{1/2} = 0.71 & \text{in the noncrowded case.} \\ 1 & \text{in the crowded case.} \end{cases}$$

An alternative margin methodology?

Prelude: Standard (member by member) margin methodologies base margins on the tail risk in a trade portfolio.

1. A standard tail risk measure is value-at-risk (VaR).
2. VaR is often calculated by the "delta-normal method" (Jorion, 2007, p. 260).

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Claim: *Margin*(A) is the "aggregate" approach extrapolated from existing member by member approaches.

An alternative margin methodology?

1. Homogeneity of degree one of mean(A) and std(A) implies that *Margin*(A) naturally decomposes across members (Euler's homogeneous function theorem).
   1.1
   $$mean(A) = \sum_j \sqrt{\frac{1}{2\pi}}\sigma_j$$
   1.2
   $$std(A) = \sum_k \sigma_k \frac{\partial std(A)}{\partial \sigma_k} = \sum_k \sigma_k \sum_l \frac{1}{std(A)} \left(\frac{\pi - 1}{2\pi}\right) \sigma_l M(\rho_{kl})$$

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2. Therefore Margin(A) equals,

$$\sum_k \sigma_k \left(\underbrace{\sqrt{\frac{1}{2\pi}} + \frac{\alpha}{std(A)}\left(\frac{\pi-1}{2\pi}\right)\sigma_k}_{\text{Member-specific part ("old")}} + \underbrace{\sum_{l\neq k} \frac{\alpha}{std(A)}\left(\frac{\pi-1}{2\pi}\right)\sigma_l M(\rho_{kl})}_{\text{Crowded-trade part ("new")}} \right).$$

An alternative margin methodology?

1. To identify risk factor(s) on which members' trades crowd, the following results are useful:

   1.1
   $$\frac{\partial}{\partial \sigma^f} \mathrm{E}(A) = \sum_j \sqrt{\frac{1}{2\pi}} \frac{\sigma_f}{\sigma_j} B_{jj}$$

   1.2
   $$\frac{\partial}{\partial \sigma^f} std(A) = \left(\frac{\pi - 1}{4\pi}\right) \frac{\sigma^f}{\sigma^A} \sum_{k,l} \Big[M'(\rho_{kl}) B_{kl} +$$
   $$+ \frac{\rho_{kl}^2}{\pi - 1} \left(1 - 2\sqrt{1 - \rho_{kl}^2}\right) \left(\frac{\sigma_l}{\sigma_k} B_{kk} + \frac{\sigma_k}{\sigma_l} B_{ll}\right) \Big]$$

   with
   $$B_{kl} := n_k{}' \beta \beta' n_l \quad \text{and} \quad \beta = \mathrm{cov}(R, r^f)/\mathrm{var}(r^f)$$

An alternative margin methodology?

1. The sensitivity of $Margin(A)$ to a particular risk factor is naturally described by the following elasticity:

$$e_{\sigma_f}^{Margin(A)} = \frac{\sigma_f}{Margin(A)}\left(\frac{\partial}{\partial\sigma^f}\mathrm{E}(A) + \alpha\frac{\partial}{\partial\sigma^f}std(A)\right).$$

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Data

1. A European Multilateral Clearing Facility (EMCF) sample of “trade reports” filed by its (anonymized) members.

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5. Sample consists of 1.4 million trades by 57 clearing members in 242 securities across 228 days.

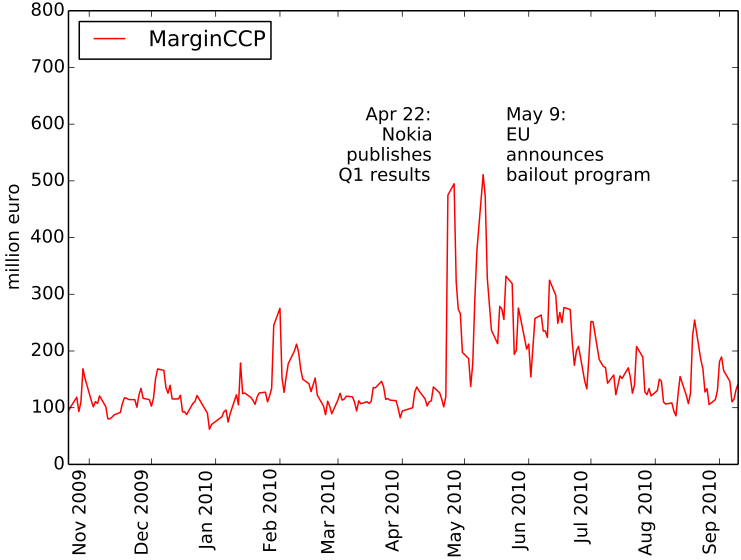
Clearing members

ABN AMRO Clearing Bank N.V.	Numis Securities Ltd
BNP Paribas Securities Services S.A.	UBS Ltd
Bank of America Merrill Lynch	Barclays Capital Securities Ltd.
Citibank Global Markets and Citibank International	Alandsbanken Abp
JPMorgan Securities Ltd.	Alandsbanken Sverige AB
Goldman Sachs International	Amagarbanken A/S
Skandinaviska Enskilda Banken	Arbejdernes Landsbank A/S
KAS BANK N.V.	Avanza Bank AB
Parel S.A.	Carnegie Bank A/S
Deutsche Bank AG	Dexia Securities France
Citigroup	E-Trade Bank
MF Global UK Ltd	Eik Bank A/S
CACEIS Bank Deutschland	EQ Bank Ltd.
Danske Bank	Evli Bank Plc
ABG Sundal Coller Norge	FIM Bank Ltd.
DnB NOR Bank	GETCO Ltd.
Deutsche Bank (London Branch)	Handelsbanken
HSBC Trinkaus & Burkhardt	Jefferies International Ltd.
Istituto Centrale delle Banche Popolari Italiane SpA	Knight Capital Markets
Interactive Brokers	Lan & Spar Bank A/S
KBC Bank N.V.	Nordnet Bank AB
Nordea	Nomura International Plc
Swedbank	Nykredit A/S
Credit Agricole Cheuvreux	Pohjola Bank
Credit Suisse Securities (europe) Ltd	RBC Capital Markets
Morgan Stanley International Plc	Saxo Bank A/S
RBS Bank N.V.	Spar Nord Bank A/S
Instinet europe Ltd.	Sparekassen Kronjylland A/S
Morgan Stanley Securities Ltd.	

Summary statistics

	Mean	Std	Min	Median	Max
Panel A: Overall summary statistics					
Daily number of reports	6,293.6	699.0	1,135.0	6,426.5	7,663.0
Daily volume (in mln shares)	160.9	42.1	8.1	155.5	342.4
Daily volume (in mln euro)	1,809.8	475.1	272.4	1,762.3	3,649.6
Volume per report (in 1000 shares)	25.6	114.1	0.0	2.6	18,631.8
Volume per report (in 1000 euro)	287.6	1,067.6	0.0	36.1	142,271.3
Panel B: Cross-sectional summary statistics, based on clearing-member averages					
Daily number of reports	114.4	143.7	0.0	64.9	736.4
Daily volume (in mln shares)	2.9	4.2	0.0	0.7	20.8
Daily volume (in mln euro)	32.9	46.9	0.0	7.8	222.4
Panel C: Cross-sectional summary statistics, based on stock averages					
Daily number of reports	26.0	21.9	0.0	20.6	84.2
Daily volume (in mln shares)	0.7	1.6	0.0	0.1	14.2
Daily volume (in mln euro)	7.5	14.6	0.0	0.9	124.0

Aggregate daily margin: actual margin and Margin(A)



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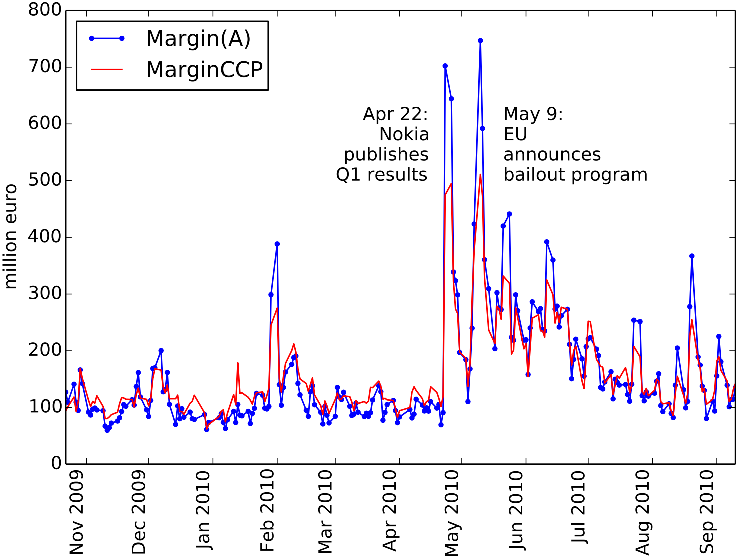

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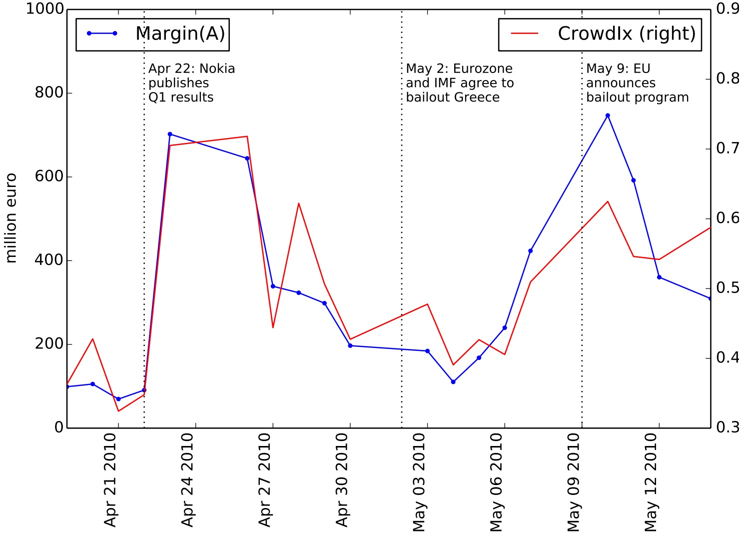
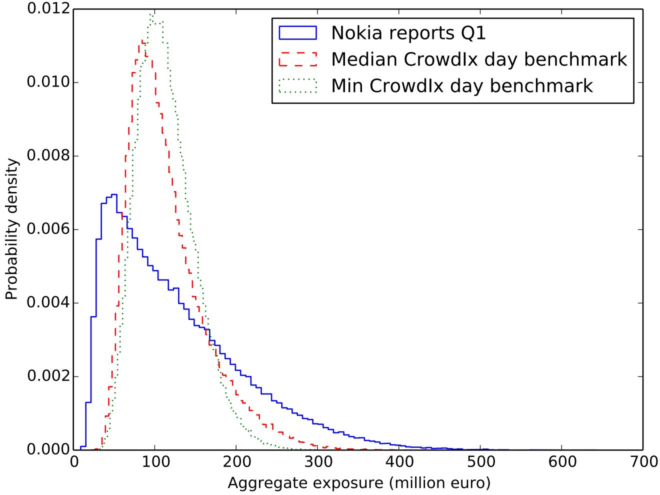
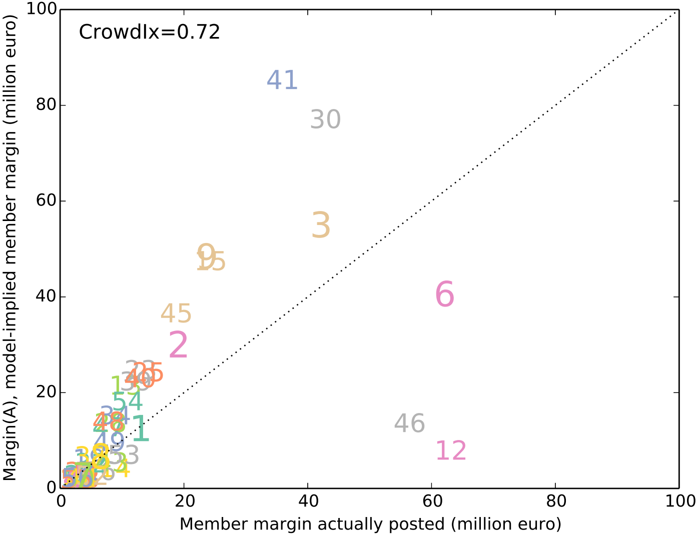


Aggregate exposure distribution “Nokia reports Q1”



Actual margin versus Margin(A)



Actual margin versus Margin(A)

Clearing member 41			
Stock	NetPos (mln €)	AbsNetPos (mln €)	AbsNetPos (%)
NOKI	-84.7	84.7	20.7
ER	64.8	64.8	15.8
FUM1V	-39.2	39.2	9.6
NDA1V	-31.7	31.7	7.7
VOLB	16.2	16.2	4.0
HMB	15.5	15.5	3.8
STERV	15.3	15.3	3.7
TLS1V	9.8	9.8	2.4
OUT1V	-8.9	8.9	2.2
SEN	-8.3	8.3	2.0

Clearing member 12			
Stock	NetPos (mln €)	AbsNetPos (mln €)	AbsNetPos (%)
VOLB	35.7	35.7	12.6
TLS1V	-17.4	17.4	6.2
MAERS	-15.2	15.2	5.4
ABBN	-13.2	13.2	4.7
ALFA	-9.7	9.7	3.4
VWS	-9.2	9.2	3.2
TRELB	-9.0	9.0	3.2
TEL2B	-8.7	8.7	3.1
ASSAB	6.8	6.8	2.4
BOLI	6.3	6.3	2.2

Margin(A) sensitivity

	Date	CrowdIx	Risk factor	Margin(A) (million euro)	ΔMargin(A) on $\Delta\sigma_f$=0.01 (million euro)	Elasticity
Median CrowdIx day	Jul 29, 2010	0.46	Market	128	81	0.91
			Nokia	128	11	0.15
			Telecom	128	46	0.46
Greek bailout	May 10, 2010	0.62	Market	747	307	0.98
			Nokia	747	27	0.14
			Telecom	747	298	0.83
Nokia reports Q1	Apr 26, 2010	0.72	Market	644	116	0.19
			Nokia	644	147	1.05
			Telecom	644	-2	-0.00

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   - 3.5 It extrapolates standard practice which should make introduction easier.
4. The implementation on real data shows that it matters, in particular when the market gets turbulent.

Crowded Risk as a Systemic Concern for Central Clearing Counterparties

Albert J. Menkveld

VU University Amsterdam and Tinbergen Institute

October 20, 2015

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Appendix A: Max crowding benchmark, $\tilde{A}$

1. If all members would trade the same risk factor, then $\exists n \in \mathbb{R}^I$ s.t. $\forall j$:
$$X_j = \nu_j \times (n' R), \quad \nu_j \in \mathbb{R}.$$
2. Then,
$$\Sigma = \underset{1\times 1}{n'\Omega n} \times \underset{J\times J}{\left(\nu_j \nu_j'\right)}.$$
3. Without loss of generality, let $n'\Omega n = 1$.
4. For member by member portfolio risks to remain unchanged, one needs $\forall j$:
$$\nu_j^2 = \sigma_j^2 \quad \Rightarrow \quad \nu_j = \pm\sqrt{\sigma_j^2}. \tag{1}$$
5. In addition, the aggregate (signed) trade is zero:
$$\sum_j \nu_j = 0. \tag{2}$$

Appendix A: Max crowding benchmark, $\tilde{A}$

1. The member trade reallocation that yields the maximum crowding benchmark is

$$\underset{\{v_1, v_2, \ldots, v_J\}}{\text{argmax}} \min\left(\sum_j v_j^+, \sum_j v_j^-\right) \text{ subject to (1),} \tag{3}$$

where

$$v_j^+ := \max(v_j, 0) \text{ and } v_j^- := \max(-v_j, 0).$$

2. If $\sum_j v_j^+ = \sum_j v_j^+$ then trade reallocation is perfect. No portfolio risk is left unallocated.

Appendix A: Max crowding benchmark, $\tilde{A}$

1. The trade reallocation is a combinatorial problem that is NP hard.
2. It maps into a one-dimensional bin packing problem (Coffman, Garey, and Johnson, 1996). Can all items be packed into two bins of size $(1/2)\sum_j \sigma_j^2$? If not, how much can be packed into two such bins? The minimum of the two bins can be matched, i.e., buyers buy this amount from sellers.
3. First fit descending (FFD) algorithm solves the offline bin packing problem in $O(J \log J)$ time (brute force requires 3^J).
4. Why FFD instead of alternative approaches?
   4.1 Average-case analysis: If item size is drawn from $U[0, 1/2]$ for one-unit bins then Coffman, Garey, and Johnson (1996, p. 39) claim "FFD is typically optimal."
   4.2 Worst-case analysis: If all items are smaller than 1/2 then FFD does as well its closest contender MFFD (modified first fit descending) (Coffman, Garey, and Johnson, 1996, p. 16-19).

Appendix B: Q&A

1. **Is it reasonable to assume equity returns are normal?** In the implementation the return distribution is assumed to be *conditionally* normal. Time-varying volatility is accounted for by calculating the covariance matrix as an exponentially weighted average of the outer product of historical daily returns.[2]

[2] EWMA(0.94) which is the RiskMetrics standard for daily equity returns.

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