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The Impact of Collateral Policies on Sovereign CDS Spreads

Giovanni Calice

Abstract

A large number of sovereign (including quasi-sovereign and parastatal) entities perform derivatives transactions on a regular basis, often through their debt management offices, but only a few post collateral on derivatives transactions. Many of them also use one-way collateral agreements in which they do not post collateral to their counterparties, but their counterparties post collateral to them. The terms for posting collateral on a derivatives transaction between a sovereign and its dealer are contained in the credit support annex (CSA) to the master agreement of the International Swaps and Derivatives Association (ISDA). During the euro area sovereign debt crisis, credit valuation adjustment (CVA) practice has emerged as one of the most important factors to have influenced the evolution of sovereign CDS (credit default swap) markets, thereby potentially exacerbating the eurozone sovereign crisis in terms of triggering both higher costs of debt and uncertainty among financial institutions on uncollateralised exposures. This paper examines the potential implications of CVA and asymmetric collateralisation on sovereign CDS spreads. Using a theoretical model, it shows that uncollateralised sovereign CDSs are subject to a collateral cost adjustment of the counterparty and that CVA enters as a driving factor altering the discounting of net swap payments. An extension of the previous studies of collateralised derivative pricing to asymmetric and imperfect collateralisation as well as to the associated CVA is provided. Approximate expressions using the Gateaux derivative allowed a straightforward numerical analysis. A pricing model shows the impact of the associated issue of collateral cost under the one-way CSA (or unilateral collateralisation), which is common when SSA (sovereign, supranational and agency) entities are involved. As a result, collateralisation may have a substantial impact on derivatives pricing through its funding effects on sovereign CDS spreads. The paper investigates the extent to which the CVA of a dealer, as a counterparty in a derivative trade, is reflected in the CDS prices of the client/sovereign counterparty. Most notably, it shows that an increase in the CVA and the cost of collateral of a dealer could translate into an increase in the price of credit protection on sovereign debt. As a result, CVA and collateral costs are priced in the CDS market: the higher the CVA of a dealer, the higher is the price at which the dealer can sell/buy credit protection on the sovereign in the CDS market. Furthermore, the magnitude of the effect could be extremely important. Some implications of this major finding for financial regulation are also discussed.

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Contents

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1. Introduction

Increased use of derivatives by financial institutions during the past couple of decades, together with a general consolidation of the international banking system have led to a structural reorganisation in the way large banks manage counterparty credit risk (CCR). Specifically, many banks have set up specialist trading units to measure and hedge CCR, known as credit valuation adjustment (CVA) desks.

Notably, the recent financial crisis, exemplified by the collapse of Lehman Brothers, has been the major driver of the regulatory push for an increased use of collateral agreements based on the credit support annex (CSA) published by the International Swap Derivatives Association (ISDA), which is now market standard among financial institutions.

Despite the significance of CCR in the financial markets, however, there has been relatively little empirical research about how it affects the prices of contracts and derivatives in which counterparties may default. This is particularly true for the $25.5 trillion notional credit default swap (CDS) market in which defaultable counterparties sell credit protection (essentially insurance) to other counterparties (BIS, 2010). CDS markets have drawn much attention in the aftermath of AIG's massive losses on CDS positions that led to the US Treasury's $182.5 billion bailout of AIG. Furthermore, concerns about the extent of counterparty credit risk in the CDS market underlie recent proposals to create a central clearinghouse for CDS transactions.[1]

Yet, the 2007-08 financial crisis has also stimulated extensive research on credit derivatives (CDs) and counterparty default risk. Financial regulators have also been working to propose a new regulatory framework for counterparty risk management and for central counterparties (CCPs) particularly in the CDS market.

Coupled with the explosion of various basis spreads, such as Libor-OIS and the cross currency swap (CCS) basis, and sovereign CDS premia, the effects of collateralisation and CVA hedging on derivative pricing, particularly from the view point of funding costs, have become important research topics. Johannes & Sundaresan (2007) were the first to emphasise the cost of collateral, using swap rates in the US market. In a series of works, Fujii & Takahashi (2009) developed a framework of interest-rate modelling in the presence of collateralisation and multiple currencies. They also stressed the importance of choice of collateral currency and the embedded cheapest-to-deliver option in collateral agreements. Piterbarg (2010) discussed the general issue of option pricing under collateralisation.

An excellent review of recent works is provided in two textbooks edited by Lipton and Rennie (2011) and Bielecki, Brigo and Patras (2011). However, the effects of collateralisation practices and CVA hedging on CDSS remain still largely unclear. A major reason for this is that

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[1] For example, see the speech by Federal Reserve Board Chairman Ben S. Bernanke at the Council on Foreign Relations on 10 March 2009. For an in-depth discussion of the economics of CDS clearinghouse mechanisms, see Duffie & Zhu (2009).

collateral costs associated with derivatives transactions have attracted a great deal of attention only recently. In this respect, the Bank of England (2010) and the International Monetary Fund (IMF, 2010) have both signalled that the current collateral treatment for bilateral CCR could have been one of the key factors behind the recent dramatic widening of sovereign CDS spreads for the most troubled euro area countries (e.g. Greece, Ireland and Portugal).

Furthermore, detailed collateral modelling is a very complex subject, due to the existence of technical specifications such as settlement lag, threshold, minimum transfer amount, etc.

The aim of this study is twofold:

First, based on the market development toward more stringent collateral management requiring a daily (or even intra-day) margin call, the paper examines the theoretical effect of asymmetric collateralisation and CVA on the pricing of sovereign CDSs under the assumption of continuous collateralisation. This is expected to be particularly relevant for the CCPs dealing with CDSs and other credit-linked products, for which the assumption of continuous collateralisation seems to be a reasonable proxy of the reality.

Second, the study aims to shed light on the interactions with the proposed capital charges under the new Basel Accord (Basel III CVA framework) and the potential additional funding costs for financial intermediaries.

The study seeks to provide a preliminary understanding of the intricate interrelationships between one-way CSA, financial institutions' CVA hedging needs – through CDSs – and the sovereign CDS market. Essentially, hardly a single sovereign entity[2] posts collateral on derivatives transactions. Moreover, many of them conduct derivatives transactions using one-way collateral agreements, such that while they may not post collateral to their counterparties, their counterparties do post collateral to them.

Activities of CVA desks for the largest derivatives dealers can have a significant impact on price discovery mechanisms in the sovereign CDS market.

Note that sovereign CDS markets are not very deep nor are they liquid. Thus, it is logical to assume that if market-makers significantly increase the volume of hedging activities, there could be substantive price impacts. Yet, it is important to stress that it is not possible to offer conclusive answers without conducting a careful empirical analysis of price impacts of different trades based on CVA trading activity of the leading dealers. Unfortunately, these data are not publicly available. Therefore, on an empirical note, such inference is plausible but not provable.

Although similar issues for standard fixed-income derivatives have been studied in previous works, the results cannot be directly applied to CD instruments since the behaviour of hazard rates generally violates the so called 'no-jump' condition (see e.g. Collin-Dufresne et al., 2004), if there exists non-trivial default dependence among the relevant parties. This work applies the technique introduced by Schonbucher (2000) and adopted later by Collin-Dufresne et al. (2004) to eliminate the necessity of this condition.

This paper contributes to an extensive literature on the effect of CCR on derivatives valuation. Important research in this area includes Cooper & Mello (1991), Sorensen & Bollier (1994), Duffie & Huang (1996), Jarrow & Yu (2001), Hull & White (2001), Longstaff (2004, 2011) and many others.

The previous works have assumed bilateral and symmetric collateralisation, where the two parties post the same currency or choose the optimal one from the same set of eligible currencies. Although symmetric collateral agreement is widely used, the asymmetric situation

[2] The well-known exceptions are Portugal (posting government bonds) and Ireland (cash collateral).

can also arise in the actual market. If there is a significant difference in credit qualities between the two parties, the relevant CSA (credit support annex) may specify asymmetric collateral treatments, such as unilateral collateralisation or asymmetric collateral thresholds. Especially, when SSA (sovereign, supranational and agency) counterparties are involved, one-way CSA is quite common. One-way CSA is now becoming a hot issue among practitioners. Since the financial firm needs to enter a two-way CSA (or bilateral collateralisation) to hedge the position in financial markets, there appears a significant cash-flow mismatch. In addition, as shown in the following sections, the financial firm may suffer from a significant loss of mark-to-market value due to the rising cost of collateral.

Asymmetric collateralisation, even if the details specified in the CSA are symmetric, may also arise effectively due to the different level of sophistication of collateral management between the two parties. For example, one party may only post single currency due to the lack of easy access to foreign currency pools or flexible operational system while the other chooses the cheapest currency each time it posts collateral.

Although, it is reasonable in normal situations to assume most of the credit exposure is eliminated by collateralisation for standard products, such as interest rate swaps, preparing for credit exposure arising from the deviation from perfect collateral coverage should be very important for risk management, particularly for complex path-dependent contracts, for which collateralisation is unlikely to achieve complete price agreements between the two parties.

This work extends previous research to more generic situations, which are asymmetric and imperfect collateralisation. The formula for the associated CVA is also derived. A generic framework, which allows asymmetry in a collateral agreement and also imperfect collateralisation, shows that the resultant pricing formula is quite similar to the one appearing in the work of Duffie & Huang (1996).

In order to see the quantitative impacts, the study analyses a CDS contract with an asymmetric collateral agreement. Although the exact solution is difficult to obtain, the Gateaux derivative allows us to obtain an approximate pricing formula for all the cases in a unified way.

Consequently, a simple pricing formula for the collateralised CDS is derived. As a result, under an asymmetric collateralisation agreement, the model suggests that the CDS price depends on the adjustment in collateral costs as well as on the associated CVA.

This is a remarkable result since the approximate pricing model clearly shows that the CDS price may be influenced by the adjustment in collateral costs as well as by the associated CVA.

Furthermore, this gives rise to very difficult questions about the potential implications of current market practices for price discovery in the sovereign CDS market. A financial institution acts as a buyer of a given CDS, by entering a back-to-back trade between the two financial firms. However, the results of the theoretical model tells us that the mark-to-market values of a sovereign CDS could significantly widen, if the financial firm adopts the conventional hedging strategy of offsetting CCR via long CDS positions in the same entity.

The findings suggest the importance of sophisticated collateral management for all financial firms. Firms carrying out an optimal collateral strategy could enjoy considerable funding benefits, while others incapable of doing so may have to pay unnecessary costs. The cost of collateral thus assumes particular importance for CVA evaluation. The present value of future credit exposure can be meaningfully modified due to the change of effective discounting rate, and can also be affected by the possible dependency between the collateral coverage ratio and the counterparty exposure. A new contribution called CCA (collateral cost adjustment) purely represents the adjustment of collateral cost due to the deviation that emerges from the perfect collateralisation.

This study therefore represents one of the first attempts to examine the impact of CVA hedging on sovereign CDSs. This contribution seeks to model the impact of unilateral derivatives collateralisation (CSA) on sovereign CDS pricing, which implies the consideration of a transmission mechanism to account for the short-term dynamics of sovereign credit risk.

Consequently, it is also a first step towards understanding whether there is a case for a current regulatory response to concerns about the potential impact posed by the CVA hedging activities of banks in these markets.

The remainder of this paper is organised as follows. Section 2 provides a brief introduction to the sovereign CDS market. Section 3 discusses CCR in the context of the CDS markets and offers an overview of the main features of collateralisation in swap contracts, based in large part on information from market surveys by ISDA. Section 4 deals with the potential causality links between CVA activity and the sovereign CDS market together with the challenges that these may pose for financial stability. Section 5 explores the implications of CVA, as a deviation from perfect collateralisation, on derivatives pricing, through the application of a generic pricing formula. The following section 6 extends the model to the pricing of CDs, focusing on CDSs and the importance of collateralisation and CVA in pricing a default swap. Within the general framework, the pricing formula for sovereign CDSs generalises a key result in Jarrow & Yu (2007). Section 7 eventually discusses conclusions.

2. The Nature of Sovereign CDS Markets

Sovereign CDSs allow investors to insure against events of default on government debt. The market for CDSs that reference advanced economy governments has grown over the past year and come under greater focus from market commentators and policymakers. .

Sovereign CDSs are similar to other CDS contracts – for example, those referencing corporate issuers.

Specifically, one counterparty (the 'protection seller') agrees to compensate another counterparty (the 'protection buyer') if the reference entity experiences a so-called credit event. For the life of the CDS contract (sovereign CDS commonly have maturities of five or ten years), the protection buyer pays the seller a premium every three months. If, however, a credit event occurs then either party can terminate the contract, prompting a payment from the seller to the buyer. This payment compensates the CDS buyer for impairments to the value of the relevant government debt. Buyers and sellers can choose to settle what they owe either using relevant sovereign debt obligations or via equivalent cash payments.

For sovereign CDS referencing advanced economies, a credit event is broadly defined as the default on, or restructuring of, a government's debt obligations. There are three principle credit events: i) failure to pay coupons or principal, ii) debt restructuring; and iii) a government official disclaiming the validity of debt obligations or imposing a moratorium or standstill, which precedes a failure to pay or restructuring

Exposures to sovereign CDSs are very small relative to the size of government bond markets. That remains the case despite a notable growth in turnover in sovereign CDSs over the past year, a period in which there has been increasing attention on public finances in a number of countries.

Sovereign CDSs are traded by a wide variety of market participants, including banks, asset management firms and hedge funds. Their motives vary. For example, hedge funds, banks and asset managers often operate on both sides of the market, selling or buying protection when they believe CDS prices are attractively high or low. For instance, UK asset managers have been selling UK sovereign CDS protection over the past months. It is also common to trade the relative prices of sovereign CDSs in different countries. For example, large banks use sovereign CDSs to hedge derivative exposures to sovereign and quasi-sovereign entities (such as central banks or supranational bodies), which do not offer collateral against changes in the value of derivative trades. In addition, some asset managers use sovereign CDSs as an approximate hedge against changes in a country's macroeconomic outlook. For example, a fund manager may seek to hedge risks on a large portfolio with exposure to bonds, equities and currencies using sovereign CDSs. This hedge does not require an event of default to prove useful – if CDS prices change, the position can be closed at a profit or loss by trading an offsetting CDS contract.

Similar to corporate CDSs, sovereign CDS prices should in principle reflect investors' perceptions of the probability of a credit event by the referenced sovereign and the expected recovery rate if this occurs. Indeed, if the possibility of default was zero, a CDS contract's price should be zero. An implied probability of default can be calculated directly from CDS prices by assuming what investors' recovery rate would be in the event of default and that investors are risk-neutral. For example, based on this simplistic approach, a five-year CDS spread of 100 basis points and a recovery rate of 40%, would give an implied (risk-neutral) probability of default that is roughly 9% over the five years.

Other factors are likely, however, to have a bearing on the price of sovereign insurance. To the extent that these factors affect the market price, they may cause default probabilities calculated in the simplistic way outlined above to be overestimated.

First and foremost, buyers of protection are likely to be risk-averse rather than risk-neutral. If so, uncertainty about the probability of default and/or the likely recovery rate in an event of default would typically increase the price of sovereign CDSs (and other types of CDSs). That is because risk-averse CDS buyers would pay extra to protect against this uncertainty.[3]

A factor particularly relevant to sovereign CDSs is the likely depreciation of the sovereign's domestic currency that would accompany a credit event. This possibility would also tend to inflate prices because sovereign CDSs are usually denominated in a different currency. Thus, the expected domestic currency pay-off is larger if the exchange rate is expected to depreciate by more.

There are also some technical issues that may influence traded CDS prices:

- The number of securities that can be used to settle CDSs may be positively related to the insurance premium because the protection buyer can choose which debt obligations are used. This option has value as the cheapest bond can be used; thus increasing the expected pay-off. The option is difficult to price, but it may be higher for sovereign CDSs than other CDSs if there are more eligible securities.
- If the creditworthiness of the protection seller and the underlying sovereign are highly correlated, there may be a low chance of the seller meeting its obligations in the event of a sovereign default. This would reduce the value of the insurance. For this reason, however, investors avoid buying sovereign protection from banks that are domiciled in the reference country.
- CDS prices may also be affected by the number of active participants and liquidity in the relevant market. This could bias traded prices either up or down.

[3] A more detailed exposition of the impact of uncertainty about default probabilities and recovery rates can be found in Pan & Singleton (2008).

Market contacts suggest that some of these factors are difficult to price and that, in practice, many traders do not explicitly take account of all of them when trading sovereign CDSs.

3. Institutional Features of Collateralisation and Counterparty Risk

3.1 Collateralisation

Collateralisation and mark-to-market (MTM) have always been an important feature of the over-the-counter (OTC) derivatives market (Litzenberger, 1992) and their use is nearly universal. There is no precise date at which MTM and collateralisation became prevalent, although there is anecdotal historical evidence that systematic collateralisation was first introduced in the late 1980s and by the early-to-mid-1990s was widespread. For example, Daigler & Steelman (1988) note that "there is not always a marking-to-market of collateral and there does not have to be any up-front margin" (p. 24), while Litzenberger (1992) and Brown & Smith (1993) note that it is common for lower-rated credits to be forced to post margin when entering into swaps with higher-rated counterparties. In 1994, in response to a demand for market-wide standards, ISDA introduced the Credit Support Annex to the Master Swap Agreement, providing a legal standard for collateralisation and facilitating the transfer of swap positions among diverse counterparties.

It is a common misperception that MTM and collateralisation became common only after the Long-Term Capital Management (LTCM) hedge fund crisis in 1998. In fact, Lowenstein (2000) noted that LTCM both collateralised and marked their positions to market: "The banks did hold collateral, after all, and Long-Term generally settled up (in cash) at the end of each trading day, collecting on winners and paying on losers" (p. 47). The main difference between LTCM and other counterparties is that LTCM refused to pay haircuts or initial margins that would have limited their ability to leverage. In a review of collateral management during the financial crisis of 1997 and 1998, ISDA (1999) reported that "many institutions avoided or greatly reduced credit losses" through collateralisation (p. 13). Moreover, the causes of any losses were not due to the inability of collateralisation to mitigate credit risk, but instead improper implementation due to inadequate haircuts (on Russian bonds) or internal data errors (omitting certain transactions).

In the last decade, collateralisation has experienced a dramatic increase in derivatives markets. ISDA (2009, 2010) market surveys indicate that collateral use is widespread and increasing. In fact, the percentage of trading volume subject to collateral agreements in the OTC market has increased from 30% in 2003 to 70% in 2010. If we focus on large broker-dealers and the fixed income market, the coverage picks up even higher (to 84%). Stringent collateral management is also a crucial issue for the successful establishment of CCPs (central clearing parties).

Discussions with market participants indicate that nearly all swaps at major investment banks are collateralised. More than 65% of "plain vanilla derivatives, especially interest rate swaps" are collateralised according to the CSA. In addition, nearly all collateral agreements are bilateral, in the sense that both counterparties post collateral if either is out-of-the money (ISDA, 2003). This is different from unilateral agreements, in which only the lower-rated counterparty posts collateral. However, in the case of sovereign CDSs, bilateral agreements are not the norm.

Due to the importance of collateralisation, new institutions such as SWAPCLEAR have been established to mitigate credit exposure through large-scale MTM and collateralisation.

ISDA (2009) notes that most of the collateral posted is in the form of USD cash (70%), US government securities (19%) or agency securities. Securities are more difficult to manage than cash as the holder must account for the risk that the value of the securities held as collateral might fall below the value of the swap. Thus, non-cash collateral is typically subject to a haircut.

An important feature of bilateral collateralisation is that interest on collateral is often rebated (see paragraph 13, item H of ISDA, 1994), in which case a short-term interest rate such as the T-bill rate is commonly paid to the collateral payer. In contrast, mark-to-market gains on futures are the property of the receiver and these gains accrue interest at the short-term rate.

The posting of collateral entails a cost and, for the other counterparty, a benefit. The easiest way to see this is to first note that the receiver of collateral reduces or eliminates any losses conditional on default. Second, collateral receivers, when it is allowed, typically reuse or rehypothecate the collateral for other purposes. Indeed, 89% of reusable collateral is rehypothecated (ISDA, 2009). Third, even when interest is rebated, there is often a cost to posting collateral as the interest rebated is typically less than the payer's funding costs. As an example, suppose that cash is posted, the cash is invested either at general collateral (GC) repo or Federal Funds rates, and T-bill rates are refunded. Due to the well-known liquidity premium embedded in Treasuries (see Grinblatt, 2001 or Longstaff, 2004), T-bill rates are lower than Federal Funds rates or GC repo rates. This difference can generate a net cost of collateral. In addition, most market participants borrow short term at rates higher than LIBOR, which entails an additional cost. Finally, another source of time-varying collateral costs is the potential for securities posted such as Treasuries to go on special, allowing the holder of special collateral to borrow at below risk-free rates. Grinblatt (2001) argues that there can be significant gains for the holders of Treasuries.

3.2 Mitigating Counterparty Credit Risk

One of the most important ways in which CCR is mitigated in the CDS market is through the market infrastructure provided by ISDA. In particular, ISDA has developed specific legal frameworks for standardised master agreements, credit support annexes, and auction, close-out, credit support, and novation protocols. These ISDA frameworks are widely used by market participants and serve to significantly reduce the potential losses arising from the default of a counterparty in a swap or derivative contract.[4]

Master agreements are encompassing contracts between two counterparties that detail all aspects of how swap and derivative contracts are to be executed, confirmed, documented, settled, etc. Once signed, all subsequent swaps and derivative transactions become part of the original master swap agreement, thereby eliminating the need to have separate contracts for each transaction.

The ISDA Master Agreement has become the industry standard for OTC derivative transactions. Under the Master Agreement, all transactions netted are documented on the basis of trade confirmations. The Master Agreement is an 'umbrella' contract governing those trade confirmations. This arrangement allows parties to an ISDA Master Agreement to aggregate all related liabilities and claims against a given counterparty, resulting in a single net amount payable by one party to the other. Reciprocal credit exposures arising from the marking to market of different OTC derivative transactions documented under an ISDA Master Agreement with the same counterparty can thus be netted against each other, which is especially beneficial when counterparties have entered into multiple transactions, such as interest rate swaps, currency swaps, equity derivatives and CDSs.

[4] Bliss & Kaufman (2006) provide an excellent discussion of the role of ISDA and of netting, collateral and closeout provisions in mitigating credit risk.

This netting feature implies that when default occurs, the market value of all contracts between counterparties A and B are aggregated into a net amount, leaving one of the two counterparties with a net liability to the other.

An important second way in which CCR is minimised is through the use of collateralisation. Recall that the value of a CDS contract can diverge significantly from zero as the credit risk of the reference firm underlying the contract varies over time. As a result, each counterparty could have a significant mark-to-market liability to the other at some point during the life of the contract.

In light of the potential credit risk, full collateralisation of CDS liabilities has become the market standard. For example, the ISDA Margin Survey[5] 2009 reports that 74% of CDS contracts executed during 2008 were subject to collateral agreements and that the estimated amount of collateral in use at the end of 2008 was approximately $4 trillion. Typically, collateral is posted in the form of cash or government securities. Participants in the Margin Survey indicate that approximately 80% of the ISDA credit support agreements are bilateral, implying two-way transfers of collateral between counterparties. Of the 20 largest respondents to the survey (all large CDS dealers), 50% of their collateral agreements are with hedge funds and institutional investors, 15% are with corporations, 13% are with banks and 21% are with others.

CSAs are standardised agreements between counterparties governing how credit risk mitigation mechanisms are to be structured. For example, a specific type of credit risk mitigation mechanism is the use of margin calls in which counterparty A demands collateral from counterparty B to cover the amount of counterparty B's net liability to counterparty A. The CSA specifies details such as the nature and type of collateral to be provided, the minimum collateral transfer amount, how the collateral amount is to be calculated, etc. ISDA protocols specify exactly how changes to master swap agreements and CSAs can be modified. These types of modifications are needed from time to time to reflect changes in the nature of the markets. For example, the increasing tendency among market participants to close out positions through novation rather than by offsetting positions motivated the development of the 2006 ISDA Novation Protocol II. Similarly, the creation of a standardised auction mechanism for settling CDS contracts on defaulting firms motivated the creation of the 2005-09 ISDA auction protocols and the 2009 ISDA close-out amount protocol.

The main features of such an arrangement are the frequency with which the net credit exposure is calculated and the minimum threshold that is to be reached before a collateral transfer is due.

Thus, it may be worthwhile for regulators to give further consideration to the feasibility and usefulness of expanding netting opportunities for banks within a legal framework across different forms of exposure. A recent development has seen netting carried out in a limited number of cases across repo and OTC derivative exposures against individual counterparties where ISDA contracts enable the pooling of collateral across these different asset classes. Being relatively new, however, this approach still faces some legal obstacles and is not deemed an alternative to the adoption of CCPs by market participants.

The real test of the robustness of this arrangement was the default of Lehman Brothers, which was a major participant in the OTC derivative market.

[5] The dominant source of the nature and extent of bilateral collateral is the ISDA's Margin Survey. ISDA has conducted this survey analysing the use of collateral as a way of mitigating credit exposures arising from OTC derivatives.

3.3 General Capital Requirements to Cover Counterparty Risk under Basel III

On 1 June 2011, the Basel Committee on Banking Supervision (BCBS) announced that it had completed its review of and finalised the Basel III capital treatment for CCR in bilateral trades. The review resulted in a minor modification of the CVA, which is the risk of loss caused by changes in the credit spread of a counterparty due to changes in its credit quality (also referred to as the market value of CCR).

Under Basel II, the risk of counterparty default and credit migration risk were addressed, but mark-to-market losses due to CVA were not. During the financial crisis, however, roughly two-thirds of losses attributed to CCR were due to CVA losses and only about one-third were due to actual defaults (Bank of England, 2010).

The Basel III framework, published in December 2010, sets out capital rules for CVA risk that include standardised and advanced methods. At the time it issued Basel III, the BCBS noted that the level and reasonableness of the standardised CVA risk capital charge was subject to a final impact assessment targeted for completion in the first quarter of 2011.

The impact study has been completed. It shows that the standardised method, as originally set out in the December 2010 rules text, could be unduly punitive for low-rated counterparties with long maturity transactions. To narrow the gap between the capital required for CCC-rated counterparties under the standardised and the advanced methods, the Basel Committee agreed to reduce the weight applied to CCC-rated counterparties from 18% to 10%.

All other aspects of the regulatory capital treatment for CCR and CVA risk remain unchanged from the December 2010 text of the Basel III rules. Overall, the Committee estimates that, with the addition of the CVA risk capital charge, the capital requirements for CCR under Basel III will double the level required under Basel II (i.e. when CCR was capitalised for default risk only). A revised version of the Basel III capital rules reflecting the CVA modification is now available.

The new Basel III standards could have significant effects on the derivatives markets and on financial institutions with large derivatives sales and trading businesses.

It is likely that the bulk of the additional capital requirements will come from the VaR on CVA.[6]

Another feature of the December 2010 BIS document is the promotion of the use of single name CDSs in order to hedge CVA. The encouragement to use single name CDSs is readily achieved via giving no capital relief for CVA hedges with the more liquid and less jumpy CDS indices. This would put CVA desks in a position where hedging may increase their required capital (and of course not hedging may reduce it).

This requirement could potentially produce some unintended consequences. Specifically, increasing capital requirements on counterparty risk provides a strong incentive for banks to

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[6] A large proportion of the Basel Committee proposals related to CCR were motivated largely by the recently discovered fact that the major component of CCR-related losses came not from actual defaults but from mark-to-market losses (according to the BIS, two-thirds of CCR losses in the crisis, although the origin of this fraction does not seem to be widely known).This leads to the proposal to charge a 'CVA VAR' against the activities of the CVA desk to capitalise their potential mark-to-market losses. Thus, to summarise, by December 2009 it had become clear that the most severe CCR related to mark-to-market losses, and Basel II had been attempting to capitalise for only one-third of CCR in the market. But the origins of this were put in place in 2005 when accountancy standards changed to require mark-to-market of CVA. Basel rules are playing catch-up with accounting rules but for some reason that process takes several years to reach even the proposal stage.

hedge out their trading exposures to sovereign entities by means of long CDS positions on the sovereign underlying reference entity.

Although this could reduce counterparty risk overall, it might also introduce systemic risks posed by the excess of demand on the sovereign CDSs themselves.

Incentivising the hedging of risk is certainly something regulators should be encouraging. But giving too much benefit for hedges that have only limited economic effectiveness (especially in turbulent markets) is possibly more dangerous than giving no benefit at all.

Regulators are stuck between a rock and a hard place here. Not allowing index hedges to generate capital relief seems unfair and counterintuitive. On the other hand, to give capital relief promotes the use of hedges that have only limited benefit and may actually fuel blow-ups. For example, consider that all banks chose to hedge the CVA of a given counterparty via an index rather than using single name CDSs (which are available but appear more expensive and less liquid). However, if the counterparty became financially distressed, CVA desks may decide to re-hedge with single name CDS contracts at a certain critical point (probably linked to the CDS spread and/or rating of the name in question). This would clearly cause a massive problem due to the likely lack of liquidity.

It may be preferable to strongly encourage hedging with single name protection during normal markets rather than to have this regime change effect in turbulent times. Single name CDSs are rather complex financial instruments that have so-called *wrong-way risk* and, through associated sudden price moves, have the ability to cause hedging problems, as noted above. Whether single name or index, a CDS is rather ubiquitous since it allows hedging of CVA but has itself potentially more toxic CVA, creating a rather difficult problem. For example, one institution might buy protection on a single-A from a double-A, then buy protection on the double-A from a triple-A, then buy protection on the triple and so on. As a result, CDS products potentially create a never-ending sequence of *wrong-way risk* and regulators shall address effectively this problem too.

4. CVA Activity and Sovereign CDS Spreads

4.1 CDS Use by Banks: Hedging and Trading Aspects

Credit risk transfer techniques can allocate risk to investors with differing time horizons, liability structures and risk tolerance. It is, however, important to understand, given the existing capital adequacy framework, how and why banks use these instruments. It is also important to investigate their effects in terms of the risk profile and the nature and type of risk in the financial system as a whole.

This section outlines the way in which CDSs interact with the existing capital framework for the regulation of banks and explains why banks use CDSs and other structured credit instruments to manage their balance sheets and hedge credit risk exposures.[7]

Clearly distinguishing the use of CDSs for hedging purposes from their use as a synthetic credit trading instrument is difficult for various reasons. The classification of CDSs as one or the other depends on the nature of the underlying asset exposure and where the CDS is booked.

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[7] A cross-jurisdictional review of regulatory capital and the treatment of CDS protection on bank loans in terms of disclosure was provided in the Joint Forum's "Report on Credit Risk Transfer" (2005) and the updated report published in April 2008.

This is complicated by the lack of sufficient comparable data for banks with similar business models. It is therefore better to regard banks' use of CDSs as part of a continuum, rather than as an 'either … or' situation.

The current crisis has shown that these neat distinctions employed in order to allocate exposures to various regulatory categories fail to reflect some of the risks incurred by banks in running their businesses. For example, there is a discrepancy between economic and supervisory capital in the case of structured credit products, indicating that regulatory capital requirements may have been too low for credit exposures, especially if they were held in the trading book.

This excessive discrepancy was recognised by the Basel Committee, which recently changed the treatment of structured credit positions held in the trading book, reducing the favourable capital treatment they have received until now.

A recent study (Minton et al., 2008) has found that, in the US, only a small number of large banks use CDs at all and that these banks use them not for hedging, but for trading. A different study, based on data on individual loans granted by a sample of US banks, found that obtaining additional credit protection through CDs made banks increase their credit supply to varying degrees, depending on the characteristics of the loans (see Hirtle, 2007).

In Europe, banks became focused on managing their balance sheets and improving returns through the use of risk transfer techniques, with CDs being one of the building blocks for balance sheet structures such as synthetic CDOs. One of the advantages of using synthetic structures is that the legal and documentary considerations associated with true-sale securitisation are lessened considerably. One further benefit from banks' point of view may be tax considerations, although this aspect may also carry with it disadvantages from a public policy perspective.

In broad terms, European banks use CDSs in a variety of ways and tend to use them for a combination of purposes, including: the hedging of both loan and cash bond exposures; risk management and the mitigation of credit risk as regards regulatory capital requirements; and trading purposes. The extent, nature and importance of these various activities in terms of banks' balance sheets and the impact on profits and losses cannot presently be determined quantitatively on a pan-European basis.

Thus it is important to collect further data and analysis in order to ensure that banks are not replacing one type of risk (i.e. credit risk) with another – counterparty – risk. In particular, further studies of balance sheet transactions and active credit portfolio management techniques are warranted, looking at how these types of active credit management interact with banks' risk profiles and IFRS accounting standards (see Prato, 2006).

4.2 CVA Definition

The CVA is the theoretical cost of replication of CCR. It was introduced for pricing purposes in Sorensen & Bollier (1994) and developed in Brigo & Masetti (2006) and Brigo & Pallavicini (2007) for interest rate swaps and exotics underlyings. Leung & Kwok (2005) and Brigo & Chourdakis (2009) worked on counterparty risk for credit (CDS) underlyings.

Back to back borrowing and lending market-indexed rates is in part how financial intermediaries operate.

The economic value of a portfolio of derivatives can be expressed as the difference between a risk-free value and the CVA of the portfolio. The CVA is the reduction in the risk free value of the portfolio due to the CCR. Actively hedging CCR is thereby transformed into actively hedging the CVA of the portfolio.

Dealers play an important role in OTC derivative markets, acting both as prime brokers, taking on counterparty risk, structuring products and providing liquidity. Market liquidity is a general precondition for market efficiency, and a sudden worsening of market liquidity may degenerate into a systemic crisis.

The interaction between liquidity (often measures by the difference in bid-offer rates), CVA and risk premia (usually measured in credit spreads) is therefore implicit in the financial system. The direction (i.e. positive or negative) and magnitude of the information spillover can be informative for policy makers in assessing the changing impact of counterparty risk in the market.

Most of the existing literature is neglecting the cost of collateral for the calculation of CVA, which seems inappropriate considering the significant size and volatility of the funding costs of holding collateral.

4.3 The role of CVA Desks

A commercial bank's CVA desk centralises the institution's control of counterparty risks by managing counterparty exposures incurred by other parts of the bank. For example, a CVA desk typically manages the counterparty risk resulting from a derivative transaction with another financial institution (such as entering an interest rate swap agreement).

The main role of the CVA desk is to consolidate credit risk management within the company. This can improve risk control procedures, including taking account of any offsetting positions with the same counterparty (which can reduce the need to hedge). CVA desks will charge a fee for managing these risks to the trading desk, which then is most likely passed on to the counterparty through the terms and conditions of the trading contract. But CVA desks are not typically mandated to maximise profits, focusing instead on risk management.

CVA desks have to operate under a mind boggling set of circumstances. First, they manage a cross-asset credit contingent book containing vanilla, exotic and highly structured trades.

Second, they trade positions on only one side of the market, due to the fact that they exclusively sell credit protection to their (internal) clients and are unable to reject transactions outright or price themselves out of a trade, nor will they be able to readily seek trades that offset the risks they take. Third, they must understand the impact of all risk mitigants, such as netting and collateral, and quantify their impact correctly. Fourth and finally, hedging of CVA is highly challenging with large transaction costs and many sensitivities which simply cannot be hedged at all. In summary, a CVA desk has a highly complex set of difficult-to-hedge risks, and operates mainly on one side of the market, which creates the constant worry of the crowded trade effect.

4.3.1 CVA Desks' Hedging of Derivatives Exposures

The pricing and hedging of CVA is progressing rapidly. Many investment banks are setting up, or have already, 'CVA desks' dedicated to the internal allocation and management of a firm's entire CCR across all products. Whilst banks and other financial institutions are at very different stages in such developments and are pursuing differing approaches, the practice of having a cross asset CVA group is emerging as a standard. CVA is being actively hedged across all asset classes, in particular via CDs and volatility positions.

In a derivative transaction, a bank may incur a loss if its counterparty defaults. Specifically, if the bank's derivative position has a positive marked-to-market (MTM) value (calculated for the remaining life of the trade) when the counterparty defaults this is the bank's 'expected positive exposure'. These potential losses are asymmetric. If the value of a bank's derivative position increases (i.e. the bank is likely to be owed money by its counterparty), the potential loss in the event of default of the counterparty will rise. In contrast, if the value of the bank's derivative position falls such that it is more likely to owe its counterparty when the contract matures then the potential loss on the transaction is zero.

Having aggregated the risks, CVA desks often buy CDS contracts to gain protection against counterparty default.[8] If liquid CDS contracts are not available for a particular counterparty, the desk may enter into an approximate hedge by purchasing credit protection via a CDS index and increase the fee charged to the trading desk to reflect the imperfect nature of the hedge. On occasion, when CDS contracts do not exist, CVA desks may try to short sell securities issued by the counterparty (i.e. borrow and then sell the securities) but this is rare.

Another way to mitigate counterparty risk is for parties to a derivative trade to exchange collateral when there are changes in the MTM value of the derivative contract. The terms of the collateral agreements between the counterparties (detailed in the credit support annex in the derivative documentation) include details such as frequency of remargining. Since MTM exposure for the bank is greatest if counterparties do not post collateral, CVA desks have reportedly been influential in promoting better risk management via tighter collateral agreements in order to reduce the CVA charge.

4.3.2 CVA Activity and the Sovereign CDS Market

Against the background of heightened investor awareness of sovereign risk, the cost to insure against default on government bonds through CDSs has risen recently. According to the Bank of England and the International Monetary Fund (IMF),[9] increased hedging by CVA desks has been an influential factor behind these moves.

Specifically, CVA desks of banks with large uncollateralised foreign exchange and interest rate swap positions with supranational or sovereign counterparties have reportedly been actively hedging those positions in sovereign CDS markets. For example, for dealers that have agreed to pay euros to counterparties and receive dollars, a depreciation of the euro will result in a MTM profit and hence a counterparty exposure that needs to be managed. As explained in the box (pp. 8–9) Bank of England (2010), given the relative illiquidity of sovereign CDS markets a sharp increase in demand from active investors can bid up the cost of sovereign CDS protection. The report also suggests that CVA desks have come to account for a large proportion of trading in the sovereign CDS market and so their hedging activity has reportedly been a factor pushing prices away from levels solely reflecting the underlying probability of sovereign default.

We should note that CDS prices will never reflect solely the probability of sovereign default because risk takers require compensation for other aspects also (a so-called risk premium). Furthermore, an increase in the cost of CDS protection cannot be straightforwardly linked to an imbalance of supply and demand in the market since it may simply be a natural reaction to a perceived increase in default risk. However, a sharp and technically driven change in a risk premium is quite plausibly indicative of a less than liquid market which should be of concern for regulators.

The Greek sovereign debt crisis provided a reminder that supposedly risk-reducing hedging activity might eventually lead to an overall detrimental effect on the market due to herd-like

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[8] The Bank of England reports that, as a consequence of the eurozone debt crisis, this activity has dramatically increased.

[9] See, for more details, the IMF Global Financial Stability Report (IMF, 2010b).

behaviour that may create market dislocations and systemic events. Hence it may be a time to question the very decisions that have led to the development and activities of CVA desks.

The analysis will be a complex chain of cause and effect where relatively benign and perhaps commonsense decisions by regulators may ultimately lead to events that are highly detrimental to the stability of financial markets acting to increase, rather than reduce, systemic risk.

Moreover, the hedging of CVA is highly problematic at the best of times. For example, for the majority of counterparties, there is no single name CDS market and hence the primary risk of CCR (default of one's counterparty) cannot even be traded. Banks will then have to mark and hedge their CVA books using proxy CDS prices and indices. The economic benefit of such hedges may be highly limited, especially in turbulent markets. Mark-to-market of CVA may fuel market instability

4.4 Financial Channels Can Amplify Sovereign Risks

Insufficient collateral requirements for sovereign counterparties in the OTC swap market can transmit emerging concerns about the credit risk of a sovereign to its counterparties. In contrast to most corporate clients, highly rated sovereign entities are generally not required to post collateral on swap arrangements.[10] Dealers may attempt to create synthetic hedges for this counterparty risk by selling assets that are highly correlated with the sovereign's credit profile, sometimes using short CDSs (so-called 'jump-to-default' hedging).

This hedging activity from uncollateralised swap agreements can put heavy pressure on the sovereign CDS market as well as other asset classes. For instance, heavy demand for jump-to-default hedges can quickly push up the price of short-dated CDS protection.

With bond dealers also trying to offset some of the sovereign risk in their government bond inventory, many European sovereign CDS curves departed from their normal upward sloping configuration to significant flattening or outright inversion. Greece's sovereign CDS curve inverted in mid-January 2011 as the funding crisis accelerated and jump-to-default hedging demand increased; Portugal's CDS curve inverted two weeks later. These pressures can easily spill over into the domestic bond market and push yields higher.

Yet sovereign CDS markets are still sufficiently shallow, especially in one-year tenors, that a large gross notional swap exposure may prompt a dealer to look to other, more liquid asset classes for a potential hedge for its exposure to sovereigns.[11] Proxies such as corporate credit, equities, or even currencies are commonly used, putting pressure on other asset classes. If swap arrangements with sovereigns were adequately collateralised, there would be less need for such defensive hedges and there would be less potential for volatility to spread from swaps to other markets.[12] However, steps to reduce transmission channels should avoid interfering with efficient market functioning and good risk management practices. Thus, recent proposals to ban 'naked' CDS exposures could be counter-productive, as this presupposes that regulators can arrive at a working definition of legitimate and illegitimate uses of these products.

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[10] Collateral requirements represent the most commonly used mechanism for mitigating credit risk associated with swap arrangements by offsetting the transactions mark-to-market exposure with pledged assets.

[11] Gross sovereign default protection is $2 trillion in notional value, just 6% of the $36 trillion global government bond market. The more relevant net exposure (true economic transfer in case of default) represents only 0.5% of government debt, at $196 billion notional amount.

[12] There is also potential for stricter collateral requirements among dealers, and between dealers and monoline insurers, and highly rated corporates and banks.

One of the most common concerns generally reported by EU banks is counterparty risk. Three conclusions can be drawn. First, large concentrated pockets of counterparty risk within the financial system cannot be assessed using aggregate data, since the data available are not broken down to the level of specific counterparts.

Additional disclosures by individual institutions on their largest exposures – in terms of counterparties and instruments, and for amounts both bought and sold – would be necessary in order to carry out such analysis.

The counterparty exposure that remains after collateralisation represents genuine counterparty risk. However, it remains very difficult to quantify this counterparty risk given the nature of the data available.

4.5 Basel III CVA Hedging and Capital Requirements Can Amplify Sovereign Risks

In an ideal world, derivatives are marked-to-market and the overall market is a zero sum game. Most OTC derivatives fall into level one or two product categorisation. Allowing banks to value such products based on their own proprietary economic models would be highly dangerous. A legitimate question that arises is the following: Since derivatives must be marked-to-market then surely their associated CVA values must be also? However, derivatives can be generally hedged via highly standardised and liquid instruments that may trade on exchanges. CVA, on the other hand, is far more complex to hedge and requires non-standard illiquid OTC instruments (and many more instruments that do not even exist). CVA appears to have become a trading book risk purely by association with the valuation of the underlying derivatives themselves. This is highly misplaced as any CVA represents a level three instrument from the point of view of valuation. Indeed, we could make a more relevant association: derivatives are exotic loans.

Since loans are not (yet) marked-to-market then neither should their CVA be. The concept that CVA can be treated on the trading book is misplaced. Even when CVA can be hedged, the market has already given us some clues as to the potential problems this causes.

Since the credit crisis, banks have been subject to strong widening in their credit spreads and, more recently, sovereigns have come under similar pressure as the perceived risk of default has risen. At the same time, the interest rate swap market has faced sharp falls in rates. Together, these two factors have significantly increased the CVA that banks face. A typical dealer's natural position from corporate and sovereign counterparties results in a long dated receiver swap and swaption exposure. Other exotic interest rate products such as CMS floors and accrual swaps also tend to contribute to this position. The majority of such risks are long dated, with the 10/30 (10-year to 30-year) part of the swap curve being highly significant.

One problem of CVA hedging is the linkage between different parameters. For example, a falling interest rate environment will increase a dealer's exposure, requiring more credit hedging, and increases in CDS spreads will lead to a need to re-hedge interest rate exposure.

Such re-hedging is required even if interest rates and CDS spreads are independent but if they are correlated then the impact is made worse (often referred to as negative gamma).

Finally, the linkage both of interest rates and of CDS spreads to long dated volatility adds a third dimension to the problem. The position then held by all dealers has the potential to cause a huge issue in a volatile market through hedging inducing feedback effects. Panic driven re-hedging tends to be accompanied by deteriorating liquidity which exacerbates the problem still further. In normal markets, rates, credit and volatility may operate more or less independently of one another but, in volatile markets, this structural connection has the potential to make them interlocked for non-economic reasons.

In May 2010, sovereign CDS spreads rose, causing significant hedging requirements for CVA desks. The resulting hedging (and front running of hedging) caused a feedback loop where such spreads, the 10/30 euro swap curve and long dated volatility all became inextricably linked. Sovereign CDS spreads widened substantially, the 10/30 swap curve flattened to below 12 bps and there was an associated increase in long dated volatility.

CVA hedging served to increase CDS spreads, drive rates down further and increase volatility, reinforcing the need to hedge further. A widening of CDS spreads automatically drove 30y rates down, made the 10/30 curve flatter and long dated volatility higher. The fact that the hedging needs of CVA desks are one way makes the problem worse. CVA desks are then forced to re-hedge at the worst levels, crossing bid and offer prices during times of rapid moves and market illiquidity. A way to avoid such problems is not to re-hedge (assuming this is within the limits structure of the CVA desk) but this makes an implicit bet on mean reversion of market parameters which, if incorrect, is embarrassing.

Strong market moves are difficult to disentangle from the natural reaction of markets to bad news. For example, CDS spreads widening dramatically, as in the case of Bear Sterns, may be simply a natural reaction to a perception of increased default probability. However, the magnitude of the Sovereign problem can be illustrated by the realised correlations between the main iTraxx indices of credit spreads and the 10/30 euro swap curve which jumped to 80% from a historical range of -30% to +30%. A similar result was found when measuring the correlation between CDS spreads and long-dated EUR interest rate volatility.[13]

Hedging CVA is a new area and traders may be prone to overreaction. Markets prone to blow ups due to their structural nature and associated re-hedging effects cannot be avoided altogether. Many markets experience granular flows due to re-hedging caused when specific thresholds are breached. Sudden thinning of liquidity, volatility increases and gaps cannot be avoided completely. The market may have to bear CVA hedging problems or improve the liquidity or variety of CD products for effective risk transfer. However, the sheer complexity of CVA hedging and its cross-asset nature suggest that the question as to whether or not hedging of CVA is beneficial at all is one that must be considered carefully.

4.6 Related Literature

There is a large and rapidly growing literature on the valuation of CCR in CDS contracts which is far too extensive for us to review fully here. Gregory (2010) provides an excellent summary of the literature and discusses a number of the modelling approaches that have been applied to the problem of valuing CCR

A number of papers have explored the theoretical magnitude of CCR on the pricing of interest rate swaps. Important examples of this literature include Cooper & Mello (1991), Sorensen & Bollier (1994), and Duffie & Huang (1996). Typically, these papers find that since the notional amount is not exchanged in an interest rate swap, the effect of CCR on an interest rate swap is very small, often only a basis point or two.

In most of the existing literature, collateral cost has been neglected, and only the reduction of counterparty exposure has been considered. The work of Johannes & Sundaresan (2007) was the first that focused on the cost of collateral and which studied its effects on swap rates based on empirical analysis. As a more recent work, Piterbarg (2010) discussed the general option pricing using the similar formula to take the collateral cost into account.

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[13] Crowded trades can cause extreme movements in both directions. The EU/IMF bailout package announced on 10 May 2010 resulted in CDS spreads tightening, a re-steepening of the 10/30 euro swap curve and lower implied volatility.

In a series of works, Fujii, Shimada & Takahashi (2009b) and Fujii & Takahashi (2011) model interest rate term structures under collateralisation, where cash collateral is assumed to be posted continuously and hence the remaining CCR is negligibly small. The authors find that there exists a direct link between the cost of collateral and CCS (cross currency swap) spreads. In fact, one cannot neglect the cost of collateral to make the whole system consistent with CCS markets, or equivalently with FX forwards.

Unlike an interest rate swap, however, a CDS contract could involve a very large payment by the protection seller to the protection buyer. For example, sellers of protection on Lehman Brothers were required to pay $91.375 per $100 notional to settle their obligations to protection buyers. Thus, the results from the interest rate swap literature may not necessarily be directly applicable to the CDS market.

Cont et al. (2009) has shown that the magnitude of financial contagion depends more on the market's network structure than the size of its largest participants. The CDS market can be regarded as an informal network of bilateral counterparty relationships and dynamic credit exposures, the size and distribution of which are closely tied to important asset markets. This chain of relationships is inherently complex and difficult to manage. In addition, the high degree of interconnectedness between market participants has also resulted in an increase in the correlation between their spreads following Lehman Brothers' failure.

The author models the diffusion of systemic risk within a network of financial institutions and assesses the impact of single-name CDSs, where CDSs introduce contingent links into the network which materialise when a credit event occurs. These new links connect the protection buyer to its counterparty, indicating the claim that would arise were a credit event to affect the reference entity. Where the protection buyer had no initial exposure to the defaulting reference entity, the credit event generates a new link in the network, the value of which could easily reach a considerable amount.

The author finds that CDSs increase both the impact of defaults by large institutions and the probability of a default having an impact, as well as systemic risk. Cont's study also underlined the fact that the ratio of speculative CDS contracts to the total amount of CDS contracts outstanding does not affect the impact in terms of systemic risk.

Consideration needs to be given to some specific aspects of the calculation of risk margins for CDSs: the 'time-varying volatility' (heteroskedasticity) of spread movements, as well as the high degree of asymmetry and large upward swings in spreads ('heavy right tails'). The potential for default in the underlying asset is a risk specific to single-name CDSs which has to be considered in CCPs' risk management. According to R. Cont, this can be done only by pooling single-name and index CDS positions in a CCP.

A few recent papers have focused on the theoretical impact of CCR on the pricing of CDS contracts. Important examples of these papers include Jarrow & Yu (2001), Hull & White (2001), Brigo & Pallavicini (2007), Kraft & Steffensen (2007), Segoviano & Singh (2008) and Blanchet-Scalliet & Patras (2008). In general, estimates of the size of the effect of CCR in this literature tend to be orders of magnitude larger than those in the literature for interest rate swaps. For example, estimates of the potential size of the pricing effect range from 7 basis points in Kraft and Steffensen to more than 20 basis points in Hull and White, depending on assumptions about the default correlations of the protection seller and the underlying reference firm. Thus, this literature tends to imply CCR pricing effects does have a substantial effect.

It is crucial to recognise, however, that this literature focuses almost exclusively on the case in which CDS contract liabilities are not collateralised. As was discussed earlier, the standard market practice would be to require full collateralisation by both counterparties to a CDS contract. This would be particularly true for CDS contracts in which one counterparty was an SSA entity.

5. Swap Valuation: The Basic Model

This section develops the methodology to deal with asymmetric and imperfect collateralisation as well as remaining CCR. All technicalities are handled in a unified way by making use of the Gateaux derivative. A formula which includes a term called 'CCA', representing adjustment of collateral cost due to the deviation from the perfect collateralisation and an additional term corresponding to CVA, is derived. The formulation therefore explicitly models the possible dependency among cost of collaterals, hazard rates, collateral coverage ratio and the underlying contract value. Even if we assume that the collateral coverage ratio and recovery rate are constant, the change of effective discounting rate induced by collateral cost and its correlation to other variables may significantly change the value of CVA.

Direct link of CCS spread and collateral cost allows us to study the numerical significance of asymmetric collateralisation. From the numerical analysis using CDSs and CCS, the relevance of sophisticated collateral management is now clear. If a financial firm is incapable of choosing the cheapest collateral currency, it has to pay very expensive funding costs to the counterparty. This section also discusses the issue of one-way CSA, which is common when SSA entities are involved. If the funding cost of collateral (or ' y ') rises, the financial firm that is the counterparty of the SSA entity may suffer from significant loss of mark-to-market value as well as the huge cash-flow mismatch. Proofs are relegated to the Appendix.

5.1 Generic Formulation

The section deals with the generic pricing formula. As an extension from the previous works, asymmetric and/or imperfect collateralisation with bilateral default risk is taken into account. The paper follows the setup in Duffie & Huang (1996) and extends it so that can deal with cost of collateral explicitly. The approximate pricing formulas that allow simple analytic treatment are derived by Gateaux derivatives.

5.2 Fundamental Pricing Formula

5.2.1 Setup

A filtered probability space $(\Omega, \Im, F, Q)$, where $F = \{\Im_t : t \geq 0\}$ is sub- σ -algebra of $\Im$ satisfying the usual conditions, is considered. Here, Q is the spot martingale measure, where the money market account is being used as the numeraire. There are two counterparties, which are denoted as party 1 and party 2. The model considers the stochastic default time of party i $(i \in (1,2))$ as an - $\Im$ stopping time $\tau^i \in [0, \infty]$, which is assumed to be totally inaccessible.

For each i, a default indicator function, $H_t^i = 1_{\{\tau^i \leq t\}}$, is considered. Such indicator is a stochastic process equal to one if party i has defaulted, and zero otherwise. The default time of any financial contract between the two parties is defined as $\tau = \tau^1 \wedge \tau^2$, the minimum of τ^1 and τ^2. The corresponding default indicator function of the contract is denoted by $H_t = 1_{\{\tau \leq t\}}$. The Doob-Meyer theorem implies the existence of the unique decomposition as $H^i = A^i + M^i$, where A^i is a predictable and right-continuous (it is continuous indeed, since it is assumed a total inaccessibility of default time), increasing process with $A_0^i = 0$, and M^i is a Q-

martingale. In the following, another assumption is the absolute continuity of A^i and the existence of progressively measurable non-negative process h^i, usually called the hazard rate of counterparty i, such that

$$A_t^i = \int_0^t h_s^i 1_{\{\tau^i \succ s\}} ds, \qquad t \geq 0. \qquad (5.1)$$

The formula also assumes that there is no simultaneous default with positive probability and hence the hazard rate for H_t is given by $h_t = h_t^1 + h_t^2$ on the set of $\{\tau \succ t\}$.

Collateralisation by cash is considered and works in the following way: if the party $(i \in (1,2))$ has negative mark-to-market, it has to post the cash collateral[14] to the counterparty $j(\neq i)$, where the coverage ratio of the exposure is denoted by $\delta_t^i \in R_+$. The margin call and settlement occur instantly. Party j is then a collateral receiver and has to pay collateral rate c_t^i on the posted amount of collateral, which is $\delta_i^t \times$ (|mark- to-market|), to the party i. This is done continuously until the end of the contract. A common practice in the market is to set c_t^i as the time-t value of overnight (ON) rate of the collateral currency used by the party i. It is crucial to distinguish the ON rate $c^i{}_i$ from the theoretical risk-free rate of the same currency r^i, where both of them are progressively measurable. The distinction is necessarily for the unified treatment of different collaterals and for the consistency with cross currency basis spreads, or equivalently FX forwards in the market.

The assumption of continuous collateralisation is considered a reasonable proxy of the current market where daily (even intra-day) margin call is becoming popular. This study is mostly interested in well-collateralised situations, where $\delta_t^i \cong 1$. However, it also includes under- and over-collateralised cases, in which we have $\delta_t^i \prec 1$ and $\delta_t^i \succ 1$, respectively. Although it may look slightly odd to include the $\delta_t^i \neq 1$ case under the continuous assumption at first sight, allowing under- and over-collateralisation makes the model more realistic considering the possible price dispute between the relevant parties, which is particularly the case for exotic derivatives. Most of the long dated exotics, such as PRDC and CMS-related products, contain path-dependent knock-out or early redemption triggers, which makes the sizable price disagreements between the two parties almost inevitable. Because of the model uncertainty, the price reconciliation is usually done in ad-hoc way, say taking an average of each party's quote. As a result, even after the each margin settlement, there always remains sizable discrepancy between the collateral value and the model implied fair value of the portfolio. Therefore, even in the presence of timely margining, the inclusion of generic collateral coverage ration taking value bigger or smaller than 1 should be important for portfolios containing exotics.

Under the assumption, the remaining credit exposure of the party i to the party j at time t is given by

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[14] According to a 2010 ISDA survey, more than 80% of collateral being used is cash. If there is a liquid repo or security-lending market, we may also carry out similar formulation with proper adjustments of its funding cost.

$$\max\left(1-\delta_t^j,0\right)\max\left(V_t^i,0\right)+\max\left(\delta_t^i-1,0\right)\max\left(-V_t^i,0\right)$$

where V_t^i denotes the mark-to-market value of the contract from the view point of party i. The second term corresponds to the over-collateralisation, where the party i can only recover the fraction of overly posted collateral when party j defaults. The recovery rate of the party j, when it defaults at time t, is denoted by the progressively measurable process $R_t^j \in [0,1]$. Thus, the recovery value that the party i receives can be written as

$$R_t^j \max\left(1-\delta_t^j,0\right)\max\left(V_t^i,0\right)+\max\left(\delta_t^i-1,0\right)\max\left(-V_t^i,0\right) \tag{5.2}$$

As for notations, a bracket "$(\ )$" is used when a type of currency is specified, such as $r_{(t)}^i$ and $c_{(t)}^i$, the risk-free and the collateral rates of currency (i), to distinguish it from that of counterparty. Spot FX at time t is denoted by $f_{xt}^{(i,j)}$ that is the price of a unit amount of currency (j) in terms of currency (i). All technical conditions for integrability are satisfied throughout this setting.

5.2.2 Pricing Formula

Ex-dividend price at time t of a generic financial contract made between the party 1 and 2, whose maturity is set as $T(\succ t)$, is considered. The model also takes into account the valuation from the view point of party 1 and the cumulative dividend D_t, which is the total receipt from party 2 subtracted by the total payment from party 1. The contract value is defined as S_t with $S = 0_t$ for $\tau \le t$.[15] Under these assumptions and the setup give in Section 2.1.1, the result is

$$S_t = B_t E^Q\left[\int_{]t,T]}\beta_u^{-1}1_{\{\tau\succ u\}}\left\{dD_u + \left(y_u^1\delta_u^1 1_{\{S_u\prec 0\}} + y_u^2\delta_u^2 1_{\{S_u\ge 0\}}\right)S_u du\right\}\right.$$
$$\left.+\int_{]t,T]}\beta_u^{-1}1_{\{\tau\ge u\}}\left(Z^1(u,S_{u-})dH_u^1 + Z^2(u,S_{u-})dH_u^2\right)\Im_t\right] \tag{5.3}$$

on the set of $\{\tau \succ t\}$. Here, $y^i = r^i - c^i$ denotes a spread between the risk-free and collateral rates of the currency used by party i, which represents the instantaneous return from the collateral being posted, i.e. it earns r^i but subtracted by c^i as the payment to the collateral

[15] See Duffie & Huang (1996) for the technical details about the regularity conditions which guarantee the existence and uniqueness of S_t.

payer. $\beta_t = \exp\left(\int_o^t r_u du\right)$ is a money market account for the currency on which S_t is defined. Z_i is the recovery payment from the view point of the party 1 at the time of default of party $i(\in \{1,2\})$:

$$Z^1(t,v) = \left(1-\left(1-R_t^1\right)\left(1-\delta_t^1\right)^+\right)v1_{\{\upsilon \prec 0\}} + \left(1+\left(1-R_t^1\right)\left(\delta_t^2-1\right)^+\right)v1_{\{\upsilon \geq 0\}} \quad (5.4)$$

$$Z^2(t,v) = \left(1-\left(1-R_t^{21}\right)\left(1-\delta_t^2\right)^+\right)v1_{\{\upsilon \geq 0\}} + \left(1+\left(1-R_t^2\right)\left(\delta_t^1-1\right)^+\right)v1_{\{\upsilon \prec 0\}} \quad (5.5)$$

where X^+ denotes $\max(X,0)$. Note that the above definition is consistent with the setup in Sec.2.1.1. The surviving party loses money if the received collateral from the defaulted party is not enough or if the posted collateral to the defaulted party exceeds the fair contract value.

Even taking the cost of collateral into account, it is possible to prove the following proposition about the pre-default value of the contract in completely parallel fashion with the one given in Duffie & Huang (1996).

Proposition 1

Suppose a generic financial contract between the party 1 and 2, of which cumulative dividend at time t is denoted by D_t from the view point of the party 1. Assume that the contract is continuously collateralised by cash where the coverage ratio of the party $i(\in \{1,2\})$'s exposure is denoted by $\delta_t^i \in \Re_+$. The collateral receiver has to pay collateral rate denoted by c_t^i on the amount of collateral posted by party i, which is not necessarily equal to the risk-free rate of the same currency, r_t^i. The fractional recovery rate $R_t^i \in [0,1]$ is assumed for the under- as well as over-collateralised exposure. For the both parties, totally inaccessible default is assumed, and the hazard rate process of party i is denoted by h_t^i.No simultaneous default of the party 1 and 2 is assumed. Then, the pre-default value V_t of the contract from the view point of party 1 is given by

$$V_t = E^Q\left[\int_{]t,T]} \exp\left(-\int_t^s r_u - \mu(u,V_u)du\right)dD_s \middle| \Im_t\right], \qquad t \leq T \quad (5.6)$$

where

$$\mu(t,\upsilon) = \left(y_t^1\delta_t^1 - \left(1-R_t^1\right)\left(1-\delta_t^1\right) + h_t^1 + \left(1-R_t^2\right)\left(\delta_t^1-1\right) + h_t^2\right)1_{\{\upsilon \prec 0\}}$$
$$+\left(y_t^2\delta_t^2 - \left(1-R_t^2\right)\left(1-\delta_t^2\right) + h_t^2 + \left(1-R_t^1\right)\left(\delta_t^2-1\right) + h_t^1\right)1_{\{\upsilon \geq 0\}} \quad (5.7)$$

if the jump of V at the time of default $(=\tau)$ *is zero almost surely, and then satisfies* $S_t = V_t 1_{\{\tau \succ t\}}$ *for all* t*. Here,* S_t *is defined in Equation (2.3).*

One important point regarding to this result is the fact that it can actually determine y^i almost uniquely from the information of cross currency market.

***Remark*:**

This remark discusses the assumption of $\Delta V_\tau = 0$. Notice that, since totally inaccessible default time is assumed, there is no contribution from pre-fixed lump sum coupon payments to the jump. In addition, it is natural (and also common in the existing literature) to assume global market variables, such as interest rates and FX's, are adapted to the background filtration independent from the defaults. This paper concentrates on the standard fixed income derivatives without credit sensitive dividends, and hence the only thing to care about is the behaviour of hazard rates, h^1 and h^2.

Therefore, in this case, if there is no jump on h^i on the default of the other party $j \neq i$, then the assumption $\Delta V_\tau = 0$ holds true. This corresponds to the situation where there is no default dependence between the two firms.

If there exists a non-zero default dependence, which is important from a risk-management point of view, then there appears a jump on the hazard rate of the surviving firm when a default occurs. This represents a direct feedback (or a contagious effect) from the defaulted firm to the surviving one. In this case, using $\Im$-intensities h^i, the no-jump assumption does not hold.

5.3 CVA as a Deviation from Perfect Collateralisation

As another important application of the Gateaux derivative, CVA as a deviation from the perfect collateralisation should be considered. Most of the existing literature neglects the cost of collateral in the calculation of CVA, which seems inappropriate considering the significant size and volatility of y, as pointed out in the work of Fujii &Takahashi (2011).

5.3.1 Derivation of CVA

Let us suppose $y_t^1 = y_t^2 = y_t$ for simplicity. In this case, the results are

$$\mu(t.V_t) = y_t - \left(\left(1-\delta_t^1\right)y_t + \left(1 - R_t^1\left(1-\delta_t^1\right) + h_t^1 - \left(1-R_t^2\right)\left(\delta_t^1 - 1\right) + h_t^2\right)1\{V_t \prec 0\}\right)$$
$$- \left(\left(1-\delta_t^2\right)y_t + \left(1 - R_t^2\left(1-\delta_t^2\right) + h_t^2 - \left(1-R_t^1\right)\left(\delta_t^2 - 1\right) + h_t^1\right)1\{V_t \geq 0\}\right) \tag{5.8}$$

considering the Gateaux derivative around the point of $\delta^1 = \delta^2 = 1$. This result can be interpreted as a bilateral CVA that takes into account the cost of collateral and its coverage ratio explicitly. Also a new term 'CCA' (*collateral cost adjustment*) appears, which is purely the adjustment of collateral cost totally independent from the CCR.

Following the methodology in Duffie & Skids (1994), it can be derived that

$$\nabla V_t = E^Q\left[\int_{]t,T]} e^{-\int_t^s (r_u - y_u)du}\left(-V_s(0)\right)*\right.$$

$$\left[\left\{\left(1-\delta_s^1\right)y_s + \left(1-R_s^1\right)\left(1-\delta_s^1\right) + h_s^1 - \left(1-R_s^2\right)\left(\delta_s^1 - 1\right) + h_s^2\right\}1\left\{V_s(0) \prec 0\right\}\right.$$
$$\left. + \left\{\left(1-\delta_s^2\right)y_s + \left(1-R_s^2\right)\left(1-\delta_s^2\right) + h_s^2 - \left(1-R_s^1\right)\left(\delta_s^2 - 1\right) + h_s^1\right\}1\left\{V_s(0) \geq 0\right\}\ \right]\Im_t\ \Big] \quad (5.9)$$

where

$$V_t(0) = E^Q\left[\int_{]t,T]} \exp\left(-\int_t^s (r_u - y_u)du\right)dD_s \middle| \Im_t\right] \quad (5.10)$$

which represents the contract value under the perfect collateralisation. Using the above result, the contract value can be decomposed into three parts, one of which is the value under the perfect collateralisation, CCA (*collateral cost adjustment*) and CVA.[16]

$$V_t \cong V_t(0) + CCA + CVA \quad (5.11)$$

This decomposition would be useful for practitioners who know that most of their exposure is collateralised, but still care about the remaining small counterparty exposure and adjustment of collateral cost due to the deviation from the perfect collateralisation.[17] It is natural to expand around the perfectly collateralised limit, since it would be the only choice that can achieve the required transparency as the benchmark price in the market.

By expanding Equation 5.9, it results

$$CCA = E^Q\left[\int_t^T e^{-\int_t^s (r_u - y_u)du} y_s\left[\left(1-\delta_s^1\right)\left[-V_s(0)\right]^+ - \left(1-\delta_s^2\right)\left[V_s(0)\right]^+\right]ds \middle| \Im_t\right] \quad (5.12)$$

which is a pure adjustment of collateral cost due to the deviation from the perfect collateralisation, and independent from the credit risk.

For credit-sensitive part,

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[16] Our convention of CVA is different from other literatures by sign where it is defined as the 'charge' to the clients. Thus, $CVA_{ours} = -CVA$.

[17] One can perform the same procedures even if there exist asymmetry in collateralisation. Since we expand around symmetric limit, there also appear correction terms for asymmetry.

$$CVA =$$

$$E^{Q}\left[\int_{]t,T]} e^{-\int_t^s (r_u - y_u)du}\left(1-R_s^1\right)h_s^1\left\{\left(1-\delta_s^1\right)+\left[-V_s(0)\right]^+ +\left(\delta_s^2-1\right)+\left[V_s(0)\right]^+\right\}ds \,\middle|\, \Im_t\right]$$

$$-E^{Q}\left[\int_{]t,T]} e^{-\int_t^s (r_u - y_u)du}\left(1-R_s^2\right)h_s^2\left\{\left(1-\delta_s^2\right)+\left[V_s(0)\right]^+ +\left(\delta_s^1-1\right)+\left[-V_s(0)\right]^+\right\}ds \,\middle|\, \Im_t\right] \quad (5.13)$$

For those trades that exhibit price jumps at the time of default, such as CDSs, things need to be handled separately. Section 6 derives the pre-default value of a CDS contract. Note also that the effects of a stochastic coverage ratio as well as non-zero jump at the time of default are beyond the scope of this study.

5.3.2 Implications of Collateralisation to Price Adjustments

Although the terms in CVA are pretty similar to the usual result of bilateral CVA, the discounting rate is now different from the risk-free rate and reflects the funding cost of collateral. If there is no dependency between y and other variables, such as the hazard rate, the effects of collateralisation would mainly appear through the modification of the discounting factor. As shown by Fujii & Takahashi (2011), the change of effective discounting factor due to the choice of collateral currency or optimal collateral strategy can be as big as several tens of percentage points. This can modify the resultant CVA meaningfully. In the case of correlated y and other variables, particularly the hazard rates, there may appear a new type of 'wrong way' risk. As we will see later, y is closely related to the CCS basis spread that reflects the relative funding cost difference between the two currencies involved. Hence, y is expected to be highly sensitive to the market liquidity, and hence is also strongly affected by the overall market credit conditions. Therefore, although efficient collateral management significantly reduces the credit risk, one needs to carefully estimate the remaining credit exposures when there exists a meaningful deviation from the perfect collateralisation.

Secondly, we can also expect important effects from the stochastic coverage ratios. If the main reason for the imperfection of collateralisation comes from price disputes over exotic products, δ^i may be well regressed by market skewness, volatility level, Libor-OIS and CCS basis spreads, etc. This may create non-trivial dependence among the collateral coverage ratio, the credit exposure, and also on the funding cost of collateral. By monitoring the price disagreements, financial firms should be able to construct a realistic model of δ^i for each counterparty. It will be also useful for stress testing, allowing higher dependence among them.

Thirdly, as we have seen, there appears a new term called 'CCA' which adjusts the cost of collateral from the perfect collateralisation case. Depending on the details of contracts and correlation among the underlying variables, CCA can be as important as CVA. As can be seen from Equation 5.12, it will be particularly the case when there is significant correlation between the collateral cost y and the underlying contract value. A typical examples of the products highly correlated with y are cross-currency basis swap and probably sovereign CDS contracts and, more generally, sovereign risk-sensitive products.

As a last remark, the valuation of CVA is critically dependent on the recovery or closeout scheme in general, and the result may sometimes be counterintuitive and/or inappropriate, as clearly demonstrated by the recent work of Brigo & Morini (2010). However, in the case of a collateralised contract, the dependency on the closeout conventions is expected to be quite small. This is because the creditworthiness of both parties entering the substitution trade is largely flattened by collateralisation.

5.3.3 A Special Case for CVA

Let us consider an important situation, which is the unilateral collateralisation with bilateral default risk. Suppose the situation where only the party 2 (i.e. a financial institution) is required to post collateral due to its high credit risk relative to the party 1 (i.e. a sovereign entity). We have $\delta^1 = 0$, $\delta^2 \cong 1$, and write $y_t^2 = y_t$.

In this case, we have

$$\upsilon(t.V_t) = y_t - \left[y_t 1\{V_t \prec 0\} + \left(1-\delta_t^2\right) y_t 1\{V_t \geq 0\}\right]$$
$$-\left(1-R_t^1\right)h_t^1\left(1\{V_t \prec 0\} - \left(\delta_t^2 - 1\right) + 1\{V_t \geq 0\}\right) - \left(1-R_t^2\right)\left(1-\delta_t^2\right) + h_t^2 1\{V_t \geq 0\} \quad (5.14)$$

Taking the Gateaux derivative around the point of $\mu(t,V_t) = y_t$, we have

$$\Delta V_t = E^Q\left[\int_t^T e^{-\int_t^s (r_u - y_u)du}\left(-V_s(0)\right)\right] \times$$
$$\left[y_s 1\{V_s \prec 0\} + \left(1-\delta_2^s\right) y_s 1\{V_s \geq 0\} + \left(1-R_s^1\right)h_s^1\left(1\{V_s \prec 0\} - \left(\delta_s^2 - 1\right) + 1\{V_s \geq 0\}\right)\right]$$
$$+\left(1-R_s^2\right)\left(1-\delta_s^2\right) + h_s^2 1\{V_s \geq 0\} \;\Big]\Big|\Im_t \;\Big] \quad (5.15)$$

More specifically, assuming the same collateral and payment currency (i),

$$V_t \cong V_t(0) + CCA + CVA \quad (5.16)$$

Where

$$V_t(0) = E^{Q^{(I)}}\left[\int_{]t,T]} \exp\left(-\int_t^s c_u^{(i)} du\right) dD_s \Big| \Im_t\right] \quad (5.17)$$

and

$$CCA = E^{Q^{(i)}}\left[\int e^{-\int_t^s cu^{(i)} du} y_s^i \left\{\left[-V_s(0)\right]^+ - \left(1-\delta_s^2\right)\left[V_s(0)\right]^+\right\} ds \Big| \Im_t\right] \quad (5.18)$$

$$CVA = E^{Q^{(i)}}\left[\int e^{-\int_t^s cu^{(i)}du}\left(1-R_s^1\right)h_s^1\left\{\left[-V_s(0)\right]^+ + \left(\delta_s^2-1\right)^+\left[V_s(0)\right]^+\right\}ds\middle|\Im_t\right]$$

$$-E^{Q^{(i)}}\left[\int e^{-\int_t^s cu^{(i)}du}\left(1-R_s^2\right)\left(1-\delta_s^2\right)+h_s^2\left[V_s(0)\right]^+ds\middle|\Im_t\right] \quad (5.19)$$

If party 1 receives 'strong' currency (that is the currency with high value of $y^{(i)}$, such as USD, and also imposes stringent collateral management $\delta^2 \cong 1$ on the counterparty, it can enjoy significant funding benefit from CCA. The CVA terms are usual bilateral credit risk adjustment except that the discounting is now given by the collateral rate.

Note that this example is particularly common when an SSA (sovereign, supranational and agency) entity is involved (as party 1). For example, when the party 1 is a sovereign entity or a central bank, it does not post collateral but receives it. From the view point of the counterpart financial firm (party 2), this is a real headache. As explained in the introduction, since party 2 has to enter bilateral collateralisation when it tries to hedge the position in the market, there clearly exists a significant risk of cash-flow mismatch. In addition, although the contribution from the CVA will be negligible, there exists a big mark-to-market issue from the CCA term. Even if it is not a critical matter at the current low-interest rate market, once the market interest rate starts to go up while the overnight rate c is kept low by the central bank to support economy, the resultant mark-to-market loss for the party 2 can be quite significant due to the rising cost of collateral " y " (Remember that $y^{(i)} = r^{(i)} - c^{(i)}$).

6. CSA, CVA and the Pricing of Sovereign CDS Securities

Despite its long history in the marketplace as well as its critical role in risk management, it is only after the explosion of Libor-OIS spread following the collapse of Lehman Brothers that the effects of collateralisation on derivative pricing started to attract strong attention among practitioners. In most of the existing literature, collateral cost has been neglected, and only its relationship with a counterparty exposure has been considered.

After the collapse of Lehman Brothers, investors have been suffering from the loss of transparency of prices provided by broker-dealers, each of them quoting quite different bids and offers. This is mainly because financial institutions started to pay more attention to CCR and also because there was no consensus on the proper method of discounting of future cash flows for secured contracts with collateral agreements. However, the situation is now changing. Recently, US authorities set out proposals to force European sovereign entities to collateralise their derivatives transactions. Likewise, the European Commission is considering requiring EU registered firms to collateralise positions with all non-EU sovereigns. These initiatives are still sparse and fragmented. Ideally, the most effective solution would be the implementation of a mutually agreed set of rules enforced on both sides of the Atlantic. Nevertheless, these represent unambiguous indications of convergence of the relevant authorities to a perfectly collateralised world with standard symmetric CSA.

It should be the only possible way to a smoother functioning of financial markets, since otherwise banks as well as SSAs are both exposed to undue liquidity risk. My formulation is based on the above understanding and derives CCA and CVA as a deviation from the collateralised benchmark price.

In this section, the primary-secondary framework developed in section 5 is used to derive a general pricing formula for CDSs in which explicit pricing formulas for CCA and CVA can be derived. This formulation, although simple, captures the quantitative impact of CCA and CVA on default and the pricing of sovereign CDSs. The model allows for dependency between CDS pricing and collateral costs and associated CVA. An innovation is the inclusion of CCS and CVA processes, as an independent set of state variables, thereby capturing the interdependence among the price discovery process in the CDS market and these factors. Whether CCR is reflected in the prices of CDS contracts are then tested. Finally, the study analyses whether the pricing of CCR by dealers varies by industry and sovereigns, as would be implied by a correlation-based credit model.

6.1 Fundamental Pricing Formula

In order to study the quantitative effects of one-way CSA collateralisation and its implications on CDS pricing, we need to understand the pricing of fundamental instruments under symmetric collateralisation.

It is also necessary for the calibration of the model in the first place. One obtains detailed discussion in section 5, but this section extends the results for stochastic y spread and summarise them. It also introduces a slightly simpler CDS contract, which is actually tradable in the market, in order to show the direct link of a sovereign CDS with the cost of collateral in much simpler fashion. All the results easily follow from section 5.

Throughout this section, it is assumed that the market quotes of standard products are the values under symmetric and perfect collateralisation, which should be reasonable considering the dominant role of major broker-dealers in these products and their stringent collateral management. If it is not the case, the value of any contract becomes dependent on the portfolio to a specific counterparty, which makes it impossible for financial firms to agree on the market prices.

6.1.1 Setup

A filtered probability space $(\Omega, \Im, F, Q)$ is considered, where Q is a spot martingale measure, and $F = \{\Im_t; t \geq 0\}$ is a sub-σ-algebra of $\Im$ satisfying the usual conditions. It is further denoted the set of relevant firms as $C = \{0,1,2\ldots,n\}$ and introduced a strictly positive random variable τ^i in the probability space as the default time of each party $i \in C$. The default indicator process of each party is defined as $H_t^i = 1_{\{\tau^i \leq t\}}$ and the filtration generated by this process by H^i. The model assumes a given background filtration G containing other information except defaults and writes $F = G \vee H^0 \vee H^1 \vee \ldots \vee H^n$. Thus, it is clear that τ^i is an H^i as well as F stopping time. A further assumption is the existence of non-negative hazard rate process h_i where

$$M_t^i = H_t^i - \int_0^t h_s^i 1_{\{\tau^i \succ s\}} ds, \qquad t \geq 0 \qquad (7.1)$$

is an (Q, F)-martingale. There is no simultaneous default for simplicity.

For collateralisation, the model repeats the same setup adopted in Section (5) once again for convenience: *Consider a trade between the party 1 and 2. If the party* $(i \in (1,2))$ *has negative mark-to-market value, it has to post the cash collateral to the counterparty* $j(\neq i)$, *where the coverage ratio of the exposure is denoted by* $\delta_t^i \in R_+$. *The margin call and settlement occur instantly. Party* j *is then a collateral receiver and has to pay collateral rate* c_t^i *on the posted amount of collateral, which is* $\delta_i^t \times$ *(|mark- to-market|), to the party* i. *This is done continuously until the end of the contract. Following the market conventions, the collateral rate* c_t^i *as the time-*t *value of overnight* (ON) *rate of the collateral currency used by the party* i *is set. It is not equal to the risk-free rate* r_t, *in general, which is necessary to make the system consistent with the cross currency market. All the processes except default times, such as* $\{c^i, r, \delta^i, R^i\}$, *are assumed as adapted to the background filtration* G.

6.1.2 CDS Pricing

CDS reference name, the investor, and the counterparty, are respectively denoted as party-0, party-1 and party-2. Let us define $\tau = \tau^0 \wedge \tau^1 \wedge \tau^2$ and its corresponding indicator process, $H_t = 1_{\{\tau \le t\}}$. The investor is a protection buyer and party-2 is a seller. Under this setup, the CDS price from the view point of the investor can be written as

$$S_t = \beta_t E^Q \left[\int_{]t,T]} \beta_U^{-1} 1_{\{\tau \succ U\}} \left(dD_U + q(u, S_u) S_u du \right) \right.$$

$$\left. + \left[\int_{]t,T]} \beta_U^{-1} 1_{\{\tau \ge U\}} \left(Z_u^0 dH_u^0 + Z^1(u, S_{u-}) dH_u^1 + Z^2(u, S_{u-}) dH_u^2 \right) \Im_t \right] \right. \quad (7.2)$$

where D denotes the cumulative dividend process representing the premium payment for the CDS, and $\beta_t = \exp\left(\int_0^t r_s ds \right)$ denotes the money-market account with the risk-free interest rate. Other variables are defined as follows:

$$q(t, \upsilon) = \delta_t^1 y_t^1 1_{\{\upsilon \prec 0\}} + \delta_t^2 y_t^2 1_{\{\upsilon \ge 0\}}$$

$$Z_t^0 = \left(1 - R_t^0\right)$$

$$Z^1(t, \upsilon) = \left(1 - \left(1 - R_t^1\right)\left(1 - \delta_t^1\right)^+ \upsilon 1_{\{\upsilon \prec 0\}} + \left(1 + \left(1 - R_t^1\right)\left(\delta_t^2 - 1\right)^+\right) \upsilon 1_{\{\upsilon \ge 0\}}\right)$$

$$Z^2(t, \upsilon) = \left(1 - \left(1 - R_t^1\right)\left(1 - \delta_t^2\right)^+ \upsilon 1_{\{\upsilon \ge 0\}} + \left(1 + \left(1 - R_t^2\right)\left(\delta_t^1 - 1\right)^+\right) \upsilon 1_{\{\upsilon \prec 0\}}\right)$$

Here, $y_t^i = r_t^i - c_t^i$ is the difference of the risk-free and collateral rates relevant for the collateral currency chosen by the party-i at the time t, which represents the instantaneous return of the posted collateral. Thus the term $q(t,\upsilon)$ summarises the return (or cost) of collateral from the view point of the investor. Z^i represents the default payoff when party-i defaults first among the set $\{0,1,2\}$.

Although it is possible to follow the same procedures used in section 5 based on the arguments of Duffie & Huang (1996), a careful treatment to avoid the jump in the value process at the time of counterparty default 3 is needed. This work applies the measure change technique introduced by Schonbucher and used by Collin-Dufresne et al. (2004), which leads to the following proposition in a clear-cut way.

Proposition 1

Under the assumptions given in 7.1 and appropriate integrability conditions, the pre-default value V_t corresponding to the CDS contract specified in the Equation. (7.2) is given by

$$V_t = E^{Q'}\left[\int_{]t,T]} \exp\left(-\int_t^s \left(r_u - \mu(u,V_u) + h_u^0\right) du\right)\left(dD_s + Z_s^0 h_s^0 ds\right)\middle|\Im_t'\right] \qquad (7.3)$$

where

$$\mu(u,\upsilon) = \left(y_t^1\delta_t^1 - \left(1-R_t^1\right)\left(1-\delta_t^1\right)^+ h_t^1 + \left(1-R_t^2\right)\left(\delta_t^1 - 1\right)^+ h_t^2\right)1_{\{\upsilon \prec 0\}}$$
$$+ \left(y_t^2\delta_t^2 - \left(1-R_t^2\right)\left(1-\delta_t^2\right)^+ h_t^2 + \left(1-R_t^1\right)\left(\delta_t^2 - 1\right)^+ h_t^1\right)1_{\{\upsilon \geq 0\}} \qquad (7.4)$$

and it satisfies $S_t = 1_{\{\tau \succ t\}} V_t$ for all $t \geq 0$. Here, the 'survival measure' Q' is defined by

$$\left.\frac{dQ'}{dQ}\right|_{\Im_t} = \prod_{i=0}^{2} 1_{\{\tau^i \succ t\}} \exp\left(\int_0^t \sum_{i=0}^{2} h_s^i ds\right) \qquad (7.5)$$

and the filtration $F' = \left(\Im_t'\right)_{t \geq 0}$ denotes the augmentation of F under Q'.

Corollary 1

Assume the perfect and symmetric collateralisation with domestic currency $\left(\delta^1 = \delta^2 = 1, y^1 = y^2 = y\right)$. Then, we have

$$V_t = E^{Q'}\left[\int_{]t,T]} \exp\left(-\int_t^s \left(c_u + h_u^0\right)du\right)\left(dD_s + Z_s^0 h_s^0 ds\right)\Big|\Im_t'\right] \tag{7.9}$$

as the pre-default value of the CDS given in Eq (2.2).

Corollary 2

Assume the perfect and symmetric collateralisation but the collateral currency (j)[18] is different from the evaluation currency (i). In this case, we have

$$V_t = E^{Q'_{(i)}}\left[\int_{]t,T]} \exp\left(-\int_t^s \left(c_u^{(i)} + y_u^{(i,j)} + h_u^0\right)du\right)\left(dD_s + Z_s^0 h_s^0 ds\right)\Big|\Im_t'\right] \tag{7.10}$$

where $y^{(i,j)} = y^i - y^{(j)}$. $Q_{(i)}$ and the associated $Q'_{(i)}$ denote the measure related to the money-market account of currency (i).

Notice that the term structure of $y^{(i,j)}$, which is the spread of collateral cost between the two currencies, can easily be bootstrapped from the cross-currency swap information in the market.

It is also straightforward to derive the leading order approximation under the imperfect collateralisation, or $\delta^i \neq 1$, using the Gateaux derivative.

Corollary 3

Assume asymmetric collateralisation with domestic currency and consider the perfect collateralisation. In this case, in the leading order approximation, we have

$$V_t \approx \overline{V}_t + \left(\nabla V\right)_t^1 + \left(\nabla V\right)_t^2 \tag{7.11}$$

where

$$\overline{V}_t = E^{Q'}\left[\int_{]t,T]} \exp\left(-\int_t^s \left(c_u + h_u^0\right)du\right)\left(dD_s + Z_s^0 h_s^0 ds\right)\Big|\Im_t'\right] \tag{7.12}$$

and

[18] We use "()" to denote the currency type instead of counterparty.

$$(\nabla V)_t^1 = E^{Q'}\left[\int_t^T e^{-\int_t^{s0}(c_u+h_u)du} y_s\left\{(1-\delta_s^1)\left[-\overline{V_s}\right]^+ - (1-\delta_s^2)\left[\overline{V_s}\right]^+\right\}ds \middle| \Im_t'\right] \quad (7.13)$$

$$(\nabla V)_t^1 = E^{Q'}\left[\int_t^T e^{-\int_t^{s0}(c_u+h_u)du} (1-R_s^1)h_s^1\left\{(1-\delta_s^1)^+\left[-\overline{V_s}\right]^+ + (\delta_s^2-1)^+\left[\overline{V_s}\right]^+\right\}ds \middle| \Im_t'\right]$$

$$- E^{Q'}\left[\int_t^T e^{-\int_t^{s0}(c_u+h_u)du} (1-R_s^2)h_s^2\left\{(1-\delta_s^2)^+\left[\overline{V_s}\right]^+ + (\delta_s^1-1)^+\left[-\overline{V_s}\right]^+\right\}ds \middle| \Im_t'\right] \quad (7.14)$$

In section 5, $(\nabla V)_t^1$ has been interpreted as CCA (collateral cost adjustment) and $(\nabla V)_t^2$ as CVA. Although it still seems a reasonable interpretation, more attention must be drawn on the interpretation of $\overline{V_t}$, the CDS value under the perfect collateralisation. In fact, the remainder of the paper concentrates on such value and it shows that the understanding of this simplest situation is quite non-trivial and critical for the risk management of CDS trades.

Consequently, from Equation (7.11), we can clearly see now that the pre-default value of a CDS contract is directly influenced by changes in the cost of collateralisation (funding cost of collateral) and, more importantly, by the CVA trading activity of the financial firm.

These results have many implications for current proposals to regulate the CDS market. As one example, financial regulators and market participants may view current CDS risk mitigation techniques such as the over-collateralisation of swap liabilities and bilateral netting as largely successful in addressing CCR concerns. Thus, proposals to create a central CDS exchange may not actually be effective in reducing CCR further. This implication parallels and complements the conclusions in a recent paper by Duffie & Zhu (2009).

More importantly, the implications of this model can be empirically investigated. For instances, in cases where the market perceives a counterparty relation, one can estimate its impact on default and the pricing of a CDS using CDS prices and see whether the said effect exists. When this is confirmed, more elaborate CDS pricing models can be constructed, and their parameters can be implicitly estimated and used to price CDs.

6.1.3 The Impact of Asymmetric Collateralisation on Sovereign CDSs

The impact of collateralisation are the most significant in interest rate and long dated FX markets, where they affect various types of basis spread and also FX forward. In previous two works Fujii, Shimada & Takahasi (2009a and b) have extended the formula used in to the situation where the payment and collateral currencies are different, which is crucial to handle multi-currency products.

The key question is as follows: what is the directional effect of CVA hedging and net costly collateral on sovereign CDS premia?

Sovereign swap premia certainly increase. To see this, consider the swap from the perspective of the fixed receiver and assume that CVA is positively related to net collateral costs. When CVA increase, the swap will have a positive MTM value and the fixed receiver (the sovereign) receives collateral. Conversely, when CVA falls, the swap will have a negative MTM value. However, the fixed receiver will not have to post collateral.

Thus, intuitively, it follows that the financial firm will demand a higher amount of CDSs to compensate for the acceleration of (opportunity) costs implied by MTM and costly collateral. The model formally demonstrates that under standard assumptions on net collateral costs, swap rates and swap spreads increase. These assumptions require that the sovereign CDS premia is a positive function of the net cost of collateral and CVA.

7. Conclusion

Increased use of derivatives by financial institutions during the past couple of decades, together with a general consolidation of the international banking system has led to a structural reorganisation in the way large banks manage counterparty risk. Specifically, many banks have set up specialist trading units to measure and hedge CCR, known as counterparty valuation adjustment (CVA) desks.

Concerns about CCR were significantly heightened in early 2008 by the collapse of Bear Stearns, but then skyrocketed later in the year when Lehman Brothers declared Chapter 11 bankruptcy and defaulted on its debt and swap obligations. Fears of systemic defaults were so extreme in the aftermath of the Lehman bankruptcy that euro-denominated CDS contracts on the US Treasury were quoted at spreads as high as 100 basis points.

During the euro area sovereign debt crisis, CVAs emerged as one of the most important factors that appear to have influenced the evolution of sovereign CDS markets, thereby potentially contributing to exacerbate the eurozone sovereign crisis.

This paper examines the potential implications of CVA and asymmetric collateralisation on sovereign CDS rates. Theoretically, uncollateralised sovereign CDSs are subject to adjustments in costly collateral of the counterparty and that CVA enters as a driving factor altering the discounting of net swap payments.

To this end, a model to deal with asymmetric and imperfect collateralisation and remaining CCR is developed above. All of the issues can be handled in a unified way by making use of the Gateaux derivative. The resulting formula contains CCA that represents adjustment of collateral cost due to the deviation from the perfect collateralisation, and the terms corresponding CVA, which now contains the possible dependency among cost of collaterals, hazard rates, collateral coverage ratio and the underlying contract value. Even if we assume that the collateral coverage ratio and recovery rate are constant, the change of effective discounting rate induced by collateral cost and its correlation to other variables may significantly change the value of CVA and directly impact on a pre-default value of a CDS instrument.

Direct link of CDS spreads and collateral cost allows us to study the numerical significance of asymmetric collateralisation. From the analysis using sovereign CDSs, the relevance of sophisticated collateral management is now clear. If a financial firm is incapable of choosing the cheapest collateral currency, it has to pay very expensive funding costs to the counterparty. The issue of one-way CSA, which is common when SSA entities are involved, is also part of this study. If the funding cost of collateral (or “ y ”) rises, the financial firm that is the counterparty of the SSA may suffer from significant loss of mark-to-market value as well as the huge cash-flow mismatch.

The study also examines the extent to which the CVA of a dealer, counterparty in a derivative trade, is reflected in the CDS prices of the client/sovereign counterparty. Most notably, the theoretical model suggests that an increase in the CVA and the CCS of a dealer could map into an increase in the price of credit protection on sovereign debt. However, due to data restrictions on the CVA/MTM positions of the major financial institutions, it is not possible to quantify such an impact.

The model shows that CVA and collateral costs are priced in the CDS market; the higher the CVA of a dealer, the higher the price at which the dealer can sell/buy credit protection on the sovereign in the CDS market. Furthermore, the magnitude of the effect could be extremely important.

The pricing formulation of CDSs, therefore, could lend support to the view that sovereign CDS liabilities do require a full collateralisation approach. Indeed, the price of CDSs appears to be too small to be explained by models that assume that sovereign CDS liabilities are unsecured.

These results also have implications for current proposals about restructuring derivatives markets. For example, financial regulators and market participants may view current CDS risk mitigation techniques such as the over-collateralisation of swap liabilities and bilateral netting as largely successful in addressing CCR concerns. Thus, this suggests that attempts to mitigate CCR through alternative approaches, such as the creation of a central clearinghouse for CDS contracts, may not be as effective as might be anticipated.

Current one-way CSA market practice is misplaced due to the fact that this results in unnecessary direct higher funding costs for banks and consequential greater borrowing costs for SSAentities, as potentially reflected in rising premia on sovereign CDSs. Putting CVA in the trading book under such circumstances causes more harm than good. It would be better to give the choice for a bank to manage most of their 'illiquid' CVA together with their loan portfolios. Regulators should also carefully consider the idea of imposing a homogeneous treatment of collateralisation requirements to individual SSAs by imposing a uniform approach. This could lead to the adoption of a two-way CSA for all the sovereign entities in their derivatives transactions with financial firms. Regulators can still (through capital requirements) encourage banks to manage CVA with large counterparties on the trading book, with such transitions ultimately linked to the development of the single name CDS market. However, CVA should not be put in the trading book.

The paper also touches upon the problems and challenges in the regulation of CCR. The current regulation of CCR seems to involve a set of arbitrary and sometimes over-complex rules such as capital requirements and accounting standards that seem not to complement one another. Furthermore, the secondary impact of a regulation may be counterproductive due to a series of knock-on effects such as seen with CVA hedging.

The appropriate use of CVA by banks together with clear regulation can control CCR in both stable and volatile markets. Rather than being caught up in short-term fixes, regulators should be looking at comprehensive high level reviews of all aspects of CCR and their impact on financial markets. This may lead to regulation that is not excessively complex but is transparent and captures the key aspects. Failure to do this will encourage sizeable systemic risks that will lead to further future losses to be absorbed by taxpayers.

The results in this paper have many important implications for sovereign credit risk. Clearly, future work is needed to understand the deep reasons for the strong relation between financial institutions and systemic sovereign risk.

Thus, this study leaves several avenues open to further analysis. The lack of data on the banks' exposures and collateralised positions with SSAs has restricted the analysis to a theoretical investigation of the main issues. Unquestionably, an empirical examination is warranted.

To investigate the potential spillover effects of the one-way CSA on sovereign CDSs, multiple aspects of financial institutions' cash flow requirements need to be examined as well as the market perceptions of current cash flow and future cash flow states, especially as perception usually relates to the liquidity risk to which these institutions are subject. This would allow us to research several key aspects of the market perception of liquidity risk faced by banks. Obviously, data on uncollateralised OTC derivative exposures of sovereign entities and CVAs of the major dealers will be integral to any empirical analysis.

Three related issues require further analysis. First and foremost, net costly collateral, liquidity, and default are clearly related. For example, it might be worthwhile to extend this model along the lines of He (2001) and Collin-Dufresne & Solnik (2001) and then calibrate it against real-market data (e.g. sovereign CDS prices, government treasuries for the default-free curve, etc.).

This would allow us to separately model the liquidity and flight-to-quality components of treasuries and the default component in the CDS. Given these components, the relative contributions and relationships among liquidity, default and costly collateral could be further analysed. Second, it would be of particular interest to understand the theoretical determinants of costly collateral. If there are market participants with differing credit profiles, whose collateral costs matter?

Only further research, beyond the scope of this study, can shed light on these and other related issues.

References

Acharya, S. (2001), "*Theory of Systemic Risk and Design of Prudential Bank Regulation*", Working Paper, London Business School, London.

Aikman, D. and G. Vlieghe (2004), "How Much Does Bank Capital Matter?", Bank of England, *Quarterly Bulletin*, Spring, pp. 48-58.

Allen, F. and D. Gale (1994), *Financial Innovation and Risk Sharing*, London: MIT Press.

—————— (2006), *Systemic Risk and Regulation*, Working Paper No. 05-24, Financial Institutions Center, Wharton School, University of Pennsylvania.

Bank of England (2001), *Risk Transfer between Banks, Insurance Companies and Capital Market: An Overview*, London, December.

—————— (2008), *Financial Stability Report*, London, April.

—————— (2010), "Markets and Operations", *Quarterly Bulletin*, Vol. 50, No. 2, London.

Basel Committee on Banking Supervision (2005), *Credit Risk Transfer*, Bank for International Settlements, Basel, March.

Baur, D. and E. Joossens (2006), "The Effect of Credit Risk Transfer on Financial Stability", EUR Working Paper No. 21521.

Benink, H.A. (1998), "Financial Fragility: Theories and Empirical Evidence", in G.G. Kaufman (ed.), *Research in Financial Services – Bank Crises: Causes, Analysis and Prevention*, Vol. 10, Stamford/London: JAI Press.

Bielecki, T., D. Brigo and F. Patras (2011), *Credit Risk Frontiers: Sub-Prime Crisis, Pricing and Hedging, CVA, MBS, Ratings and Liquidity*, Bloomberg Financial Series, Hoboken, NJ: John Wiley and sons.

BIS (Bank for International Settlements) (2001), "Marrying the Macro- and Micro-Prudential Dimensions of Financial Stability", BIS Paper No. 1, Basel.

_____ (2001), "Collateral in Wholesale Financial Markets: Recent Trends, Risk Management and Market Dynamics", prepared for the Committee on the Global Financial System, Basel.

_____ (2010), "Semi-annual OTC Derivatives Statistics at End-December 2010", Basel.

Blanchet-Scalliet, C. and F. Patras (2008), "Counterparty Risk Valuation for CDS", Working Paper, University of Lyon, Lyon.

Bliss R. and G. Kaufman (2006), "Derivatives and Systemic Risk: Netting, Collateral, and Closeout", *Journal of Financial Stability*, Vol. 2, pp. 55-70.

Bomfim, A.N (2005), *Understanding Credit Derivatives and Related Instruments*, Amsterdam: Elsevier.

Brigo, D. and K. Chourdakis (2009), "Counterparty Risk for Credit Default Swaps: Impact of spread volatility and default correlation, *International Journal of Theoretical and Applied Finance*, Vol. 12, No. 7, pp. 1007-1026.

Brigo, D. and M. Masetti (2006), "Risk Neutral Pricing of Counterparty Risk", in M. Pykhtin (ed.), *Counterparty Credit Risk Modeling: Risk Management, Pricing and Regulation*, London: Risk Books.

Brigo, D. and M. Morini (2010), "Dangers of Bilateral Counterparty Risk: the fundamental impact of closeout conventions", available at arXiv.

Brigo, D. and A. Pallavicini (2007). "Counterparty Risk under Correlation between Default and Interest Rates", in J. Miller, D. Edelman and J. Appleby (eds), *Numerical Methods for Finance*, Chapman Hall.

————— (February 2008), "Counterparty Risk and Contingent CDS with Stochastic Intensity Hybrid Models", *Risk Magazine*.

Brown, K. and D. Smith (1993), "Default Risk and Innovations in the Design of Interest Rate Swaps", *Financial Management*, Vol. 22, pp. 94–105.

Calamaro, J.-P., T. Nassar, K. Thakkar and J. Tierney (2004), *Trading Tranched Index Products: The First Steps*, Deutsche Bank Global Markets Research – Quantitative Credit Strategy.

Chen, L., D.A. Lesmond and J. Wei (2007), "Corporate Yield Spreads and Bond Liquidity", *Journal of Finance*, Vol. 62, No. 1, pp. 119-149.

Clarke, M. (1999), "An Introduction to the Legal Aspects of Collateralisation", Technical Report, JP Morgan.

Collin-Dufresne, P., R.S. Goldstein and M. Spencer (2001), "The Determinants of Credit Spread Changes", *Journal of Finance*, No. 56, pp. 2177-2207.

Collin-Dufresne, P. and B. Solnik (2001), "On the Term Structure of Default Premia in the Swap and LIBOR Markets", *Journal of Finance*, Vol. 56, No. 3, pp. 1096-1115.

Collin-Dufresne ,P., R. Goldstein and J. Helwege (2002), "Are Jumps in Corporate Bond Yields Priced? Modeling Contagion via the Updating of Beliefs", Working Paper, Carnegie Mellon University, Pittsburgh PA.

Collin-Dufresne, P., R. Goldstein and J. Hugonnier (2004), "A General Formula for Valuing Defaultable Securities", *Econometrica*, Vol. 72, pp. 1377-1407.

Cont, R., A. Minca and A. Moussa (2009), "Measuring Systematic Risk in Financial Networks", Columbia University, New York, NY and Université de Paris, Paris.

Cooper, I. and A. Mello (1991), "The Default Risk on Swaps", *Journal of Finance*, Vol. 46, pp. 597-620.

Daigler, R. and D. Steelman (1988), "Interest Rate Swaps and Financial Institutions", Working Paper, Kidder Peabody, New York, NY.

Das, S.R., L. Freed, G. Geng and N. Kapadia (2006), "Correlated Default Risk", *The Journal of Fixed Income*, Vol.16, No.2, pp. 7-32.

De Bandt, O. and P. Hartmann (2002), "Systemic Risk: A Survey", in C.A.E. Goodhart and G. Illing (eds), *Financial Crisis, Contagion and the Lender of Last Resort: A Book of Readings*, London: Oxford University Press, pp. 249-298.

Delianedis, G. and R. Geske (2001), "The Components of Corporate Credit Spreads: Default, Recovery, Tax, Jumps, Liquidity, and Market Factors", Working Paper, Anderson School at UCLA, Los Angeles, CA.

Diamond, D. and R. Rajan (2001), "Liquidity Risk, Liquidity Creation and Financial Fragility: A Theory of Banking", *Journal of Political Economy*, Vol. 109, No. 2, pp. 287-327.

Dodd, R. (2004.), "Derivatives Markets: Sources of Vulnerability in US Financial Markets", in Gerald Epstein (ed.), *Financialization and the World Economy*, Cheltenham: Edward Elgar Publishers and *ICFAI Journal of Derivatives Markets*.

Dow, J. (2000), "What is Systemic Risk? Moral Hazard, Initial Shocks and Propagation", *Monetary and Economic Studies*, No. 18, pp. 1-24.

Driessen, J. (2005), "Is Default Event Risk Priced in Corporate Bonds?", *Review of Financial Studies*, No. 18, pp. 165-195.

Duffie, D. and C. Skids (1994), "Continuous-Time Security Pricing: A Utility Gradient Approach", *Journal of Mathematical Economics*, Vol. 23, pp. 107-131.

Duffie, D. and M. Huang (1996), "Swap Rates and Credit Quality", *Journal of Finance*, Vol. 51, pp. 921-949.

Duffie, D. and N. Garleanu (2001), "Risk and Valuation of Collateralised Debt Obligations", *Financial Analysts Journal*, Vol. 57, No. 1, pp. 41-59.

Duffie, D., N. Garleanu and L.H. Pedersen (2008), "Valuation in Over-The-Counter Markets", *Review of Financial Studies*, Vol. 20, pp. 1865-1900.

Duffie, D. and Z. Haoziang (2009), *Does A Central Clearing Counterparty Reduce Counterparty Risk?*, Working Paper, Stanford University, Stanford, CA.

European Central Bank (2007), *Financial Stability Review*, Frankfurt, December.

Effenberger, D. (2003), "Credit Derivatives: Implications for Credit Markets", *Frankfurt Voice*, Deutsche Bank Research.

Elton, E., M. Gruber, D. Agrawal and D. Mann (2001), "Explaining the Rate Spread on Corporate Bonds", *Journal of Finance*, No. 56, pp. 247-277.

Ericsson, J., K. Jacobs and R. Oviedo (2004), "The Determinants of Credit Default Swap Premia", Working Paper, McGill University.

Ericsson, J. and O. Renault (2006), "Liquidity and Credit Risk", *Journal of Finance*, Vol. 61, No. 5, pp. 2219-2250.

Ericsson, J., J. Reneby and H. Wang (2006), "Can Structural Models Price Default Risk? Evidence from Bond and Credit Derivative Markets", Working Paper, McGill University.

FitchRatings (2004), "Default Correlation and Its Effect on Portfolios of Credit Risk", February.

Fujii, M., Y. Shimada and A. Takahashi (2009a), "A Note on Construction of Multiple Swap Curves With and Without Collateral", CARF Working Paper Series F-154, Center for Advanced Research in Finance, University of Tokyo, Tokyo.

Fujii, M., Y. Shimada and A. Takahashi (2009b), "A Market Model of Interest Rates with Dynamic Basis Spreads in the presence of Collateral and Multiple Currencies," CARF Working Paper Series F-196, Center for Advanced Research in Finance, University of Tokyo, Tokyo.

Fujii, M. and A. Takahashi (2010), "Modeling of Interest Rate Term Structures under Collateralisation and its Implications", forthcoming in Proceedings of KIER-TMU International Workshop on Financial Engineering.

Fujii, M. and A. Takahashi (2011), "Choice of Collateral Currency", *Risk Magazine*, January, pp. 120-125.

Gibson, M.S. (2004), "Understanding the Risk of Synthetic CDOs", Finance and Economics Discussion Paper No. 36, Federal Reserve Board, Washington, D.C.

Goodhart, C.A.E., P. Sunirand and D.P. Tscomocos (2004), "A Model to Analyse Financial Fragility: Applications", *Journal of Financial Stability*, Vol. 1, No. 1.

Goodhart, C.A.E. (2005), "Financial Regulation, Credit Risk and Financial Stability", *National Institute Economic Review*, No. 192, pp. 118-127.

Gregory, J. (2010), *Counterparty Credit Risk: The New Challenge for Global Financial Markets,* John Wiley & Sons.

Grinblatt, M. (2001), "An Analytic Solution for Interest Rate Swap Spreads", *International Review of Finance*, Vol. 2, pp. 113-149.

Gyntelberg, J. (March 2005), "CDS Index Tranches and the Pricing of Credit Risk Correlations", *BIS Quarterly Review*, March, pp. 73-87.

He, H. (2001), "Modeling Term Structures of Swap Spreads", Working Paper, Yale University, New Haven, CT.

Hirtle, B. (2007), "Credit Derivatives and Bank Credit Supply", Federal Reserve Bank of New York Staff Report No. 276, New York, NY.

Houweling, P., A. Mentink and T. Vorst (2005), "Comparing Possible Proxies of Corporate Bond Liquidity", *Journal of Banking and Finance*, Vol. 29, No. 6, pp. 1331-1358.

Hull, J. and A. White (2001), "Valuing Credit Default Swaps II: Modeling Default Correlations", *Journal of Derivatives*, Vol. 8, No. 3, Spring, pp. 12-21.

IMF (2006), *The Influence of Credit Derivatives and Structured Credit Markets on Financial Stability, Global Financial Stability Report*, International Monetary Fund, Washington, D.C., April.

_____ (2010a), *Global Financial Stability Report*, International Monetary Fund, Washington, D.C., April.

_____ (2010b), *Global Financial Stability Report*, International Monetary Fund, Washington, D.C., October.

Instefjord, N. (2005), "Risk and Hedging: Do Credit Derivatives Increase Bank Risk?", *Journal of Banking and Finance*, No. 29, pp. 333-345.

Institute of International Finance (IIF) (2007), *Principles of Liquidity Risk Management,* Washington, D.C.

ISDA (International Swaps and Derivatives Association) (1994), Credit Support Annex, New York, NY.

_____ (1998), *Guidelines for Collateral Practitioners*, New York, NY.

_____ (1999), *ISDA 1999 Collateral Review*, New York, NY.

_____ (2000), *ISDA Collateral Survey 2000*, New York, NY.

_____ (2003), *ISDA Collateral Survey 2003*, New York, NY.

_____ (2009), "ISDA Margin Survey 2009", New York, NY (http://www.isda.org/c_and a/pdf/ISDA-Margin-Survey-2009.pdf).

_____ (2010), "ISDA Margin Survey 2010", New York, NY (http://www.isda.org/c and a/pdf/ISDA-Margin-Survey-2010.pdf).

_____(2010), "Market Review of OTC Derivative Bilateral Collateralisation Practices" New York, NY (http://www.isda.org/c and a/pdf/Collateral-Market-Review.pdf).

Jarrow, R.A. (2005), "Liquidity Risk and Classical Option Pricing Theory", in P. Neu and L. Matz (eds), *Mathematical Modeling of Market Liquidity Risk*, Singapore: John Wiley & Sons.

Jarrow, R.A. and P. Protter (2005), "Liquidity Risk and Option Pricing Theory", in J. Birge and V. Linetsky (eds), *Handbook of Financial Engineering,* Elsevier Publishers.

Jarrow, R.A. and F. Yu (2001), "Counterparty Risk and the Pricing of Defaultable Securities", *Journal of Finance*, Vol. 56, No. 5, pp. 1765-1799.

Johannes, M. and S. Sundaresan (2007), "The Impact of Collateralisation on Swap Rates", *Journal of Finance*, Vol. 62, pp. 383-410.

Joint Forum (2005), "Credit Risk Transfer", Basel Committee on Banking Supervision and Bank for International Settlements, Basel, March.

_____ (2008), "Credit Risk Transfer: Developments from 2005 to 2007", Basel Committee on Banking Supervision and Bank for International Settlements, Basel, April.

Krahnen, J. P. and C. Wilde (2006), *Risk Transfer with CDOs and Systemic Risk in Banking*, CFS Working Paper No. 4, Center for Financial Studies, Frankfurt am Main..

Kraft, H. and M. Steffensen (2007), "Bankruptcy Counterparty Risk and Contagion", *Review of Finance*, Vol. 11, pp. 209-252.

Laganá, M., M. Peřina, I. von Köppen-Mertes and A. Persaud (2006), *Implications for Liquidity from Innovation and Transparency in the European Corporate Bond Market*, ECB Occasional Paper No. 50, European Central Bank, Frankfurt am Main.

Leung, S.Y. and Y.K. Kwok (2005), "Credit Default Swap Valuation with Counterparty Risk", *Kyoto Economic Review*, 74(1), June.

Li, D.X. (2000), "On Default Correlation: A Copula Function Approach", *Journal of Fixed Income*, Vol. 9, No. 4pp. 43-54.

Lipton, A. and A. Rennie (2011), *The Oxford Handbook of Credit Derivatives,* Oxford: Oxford University Press.

Litzenberger, R. (1992), "Swaps: Plain and Fanciful", *Journal of Finance*, Vol. 42, pp. 403-417.

Longstaff, F. (2004), "The Flight to Liquidity Premium in US Treasury Bond Prices", *Journal of Business*, Vol. 77, pp. 511-526.

Longstaff, F.A., S. Mithal and E. Neis (2005), "Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from Credit-Default Swap Market", *Journal of Finance*, Vol. 60, pp. 2213-2253.

Longstaff, F.A. and A. Rajan (2008), "An Empirical Analysis of the Pricing of Collateralised Debt Obligations", *Journal of Finance*, Vol. 63, pp. 509-563.

Longstaff, F.A., A. Navneet and P. Gandhi (2011), "Counterparty Credit Risk and the Credit Default Swap Market", Working Paper, University of California at Los Angeles (UCLA), Anderson School of Management,. Los Angeles, CA.

Lowenstein, R. (2000), *When Genius Failed: The Rise and Fall of Long-Term Capital Management*, New York: Random House.

Merton, R.C. (1995), "A Conceptual Framework for Analyzing the Financial Environment", in B. Dwight et al. (eds), *The Global Financial System: A Functional Perspective*, Boston: MA: Harvard Business School Press.

Minsky, H.P. (1977), "A Theory of Systemic Financial Fragility", in E.I. Altman and A.W. Sametz (eds), *Financial Crises: Institutions and Markets in a Fragile Environment*, New York, NY: Wiley.

Minton, B., R.M. Stulz and R. Williamson (2008), *How Much Do Banks Use Credit Derivatives to Hedge Loans?*, Dice Center Working Paper No 2008-1, Fisher College of Business, Ohio State University, Columbus, OH.

Newman, Y.S. and M.A. Rierson (2004*), Illiquidity Spillovers: Theory and Empirical Evidence from European Telecom Bond Issuances*, Working Paper, Stanford University, Standford, CA.

Norden, L. and M. Weber (2004), "The Comovement of Credit Default Swap, Bond and Stock Markets: An Empirical Analysis", CFA Working Paper No. 2004/20, Center for Financial Studies, Frankfurt.

O'Hara, M. (2004), "Liquidity and Financial Market Stability", National Bank of Belgium Working Paper No. 55, Brussels.

Pan, J. and K. Singleton (2008), "Default and Recovery Implicit in the Term Structure of Sovereign CDS Spreads", *The Journal of Finance*, Vol. 63, No. 5, pp. 2345-2384.

Piterbarg, V. (2010), "Funding Beyond Discounting: Collateral Agreements and Derivatives Pricing", *Risk Magazine,* February.

Plender, J. (2005), "Shock of the New: A Changed Financial Landscape may be Eroding Resistance to Systemic Risk", *Financial Times*, 16 February.

Prato, O. (2006), "Better capturing risks in the trading book", *Financial Stability Review*, Banque de France, Paris.

Rajan, R.G. (2005), "Has Financial Development Made the World Riskier?", NBER Working Paper No. 11728, National Bureau of Economic Research, Cambridge, MA.

Rebonato, R. (2004), *Volatility and Correlation: The Perfect Hedger and the Fox*, 2nd Edition, London: John Wiley & Sons.

RISK.net (2011), "Dealers face funding time-bomb from one-way CSAs", RISK.net, February.

Rule, D. (2001), "The CDs Market: Its Development and Possible Implications for Financial Stability", *Financial Stability Review*, Bank of England, June, pp. 117-140.

Schinasi, G.J., R.S. Craig, B. Drees and C. Kramer (2000), *Modern Banking and OTC Derivatives Markets: The Transformation of Global Finance and its Implications for Systemic Risk*, IMF Occasional Paper No. 203, International Monetary Fund, Washington, D.C.

Schinasi, G.J. (2004), *Defining Financial Stability*, IMF Working Paper No. 04/187, International Monetary Fund, Washington, D.C.

Schonbucher, P. (2000), "A Libor Market Model with Default Risk", Working Paper, University of Bonn, Bonn.

Segoviano, M. and M. Singh (2008), *Counterparty Risk in the Over-the-Counter Derivatives Market*, IMF Working Paper No. 08/258, International Monetary Fund, Washington, D.C.

Sorensen, E. and T. Bollier (1994), "Pricing Swap Default Risk", *Financial Analysts Journal*, Vol. 50, No. 3, pp. 23-33.

Speyer, B. (2006), "Credit Derivatives – Short-Term Challenges and Long-Term Implications", *Journal of Financial Transformation*, Vol. 16, pp. 143-152.

Stulz, R.M. (2004), "Should we Fear Derivatives?", *Journal of Economic Perspectives*, No. 18, pp. 173-192.

Summer, M. (2003), "Banking Regulation and Systemic Risk", *Open Economies Review*, No. 14, pp. 43-70.

Szego, G. (2002), "Measures of Risk", *Journal of Banking & Finance*, Vol. 26, No. 7, May, pp. 1253-1272.

Tambakis, D.N. (2006), "Endogenous Market Turbulence", CERF Working Paper No. 27, University of Cambridge.

Tang, D.Y. and H. Yan (2006), "Liquidity, Liquidity Spillover, and Credit Default Swap Spreads", Technical Report, Kennesaw State University, Kennesaw, GA and University of South Carolina, Columbia, SC.

Tavakoli, J.M. (2003), *Collateralised Debt Obligations & Structured Finance: New Developments in Cash and Synthetic Securitization*, John Wiley & Sons.

Trichet, J.C. (2007), "Some Reflections on the Development of Credit Derivatives", Keynote Address at the 22nd Annual General Meeting of the International Swaps and Derivatives Association (ISDA), Boston, 18 April.

Tsomocos, D.P. (2003), "Equilibrium Analysis, Banking and Financial Instability", *Journal of Mathematical Economics*, Vol. 39, Nos. 5-6, pp. 619-655.

———— (2003), "Equilibrium Analysis, Banking, Contagion and Financial Fragility", Bank of England Working Paper No. 175, London.

Wagner, W. and I. Marsh (2006), "Credit Risk Transfer and Financial Sector Stability", *Journal of Financial Stability*, No. 2, pp. 173-193.

Wagner, W. (2007), "The Liquidity of Bank Assets and Banking Stability", *Journal of Banking and Finance*, Vol. 31, pp. 121-139.

Zhang, Y., H. Zhou and H. Zhu (2005), "Explaining Credit Default Swap Spreads with Equity Volatility and Jump Risks of Individual Firms", BIS Working Paper No. 181, Bank for International Settlements, Basel.

Zhu, H. (2004), "An Empirical Comparison of Credit Spreads Between the Bond Market and the Credit Default Swap Market", BIS Working Paper No. 16, Bank for International Settlements, Basel.

Appendix

Proof of Proposition 1* (section 6.1.2)*:

Using the Doob-Meyer decomposition, one obtains:

$$S_t = \beta_t E^Q \left[\int_{]t,T]} \beta_u^{-1} 1_{\{\tau \succ u\}} \left(dD_u + q(u, S_u) S_u du\right) \right.$$

$$\left. + \int_t^T \beta_u^{-1} 1_{\{\tau \succ u\}} \left(Z_u^0 h_u^0 + Z^1(u, S_u) h_u^1 + Z_u^2(u, S_u) h_u^2\right) du \middle| \Im_t \right] \qquad (7.6)$$

Let us define

$$\eta_t = \frac{dQ'}{dQ}\Big|_{\Im_t} = 1_{\{\tau \succ t\}} \wedge_t \qquad (7.7)$$

where

$$\wedge_t = \exp\left(\int_0^t h_s ds\right) \qquad (7.8)$$

and $h_s = \sum_{i=0}^{2} h_s^i$. Then, as a result,

$$S_t = \beta_t E^Q \left[\int_{]t,T]} (\beta_u \wedge_u)^{-1} \eta_u \left(dD_u + q(u, S_u) S_u du\right) \right.$$

$$\left. + \int_t^T (\beta_u \wedge_u)^{-1} \eta_u \left(Z_u^0 h_u^0 + Z^1(u, S_u) h_u^1 + Z^2(u, S_u) h_u^2\right) du \middle| \Im_t \right]$$

$$= 1_{\{\tau \succ t\}} E^{Q'} \left[\int_{]t,T]} \frac{\beta_t \wedge_t}{\beta_u \wedge_u} \left\{ dD_u + \left(q(u, S_u) S_u + Z_u^0 h_u^0 + Z^1(u, S_u) h_u^1 + Z^2(u, S_u) h_u^2\right) du \right\} \middle| \Im'_t \right]$$

Thus, on the set $\{\tau \succ t\}$,

$$V_t = E^{Q'}\left[\int_{]t,T]} \frac{\beta_t \wedge_t}{\beta_u \wedge_u} \left\{ dD_u + \left(q(u,V_u) V_u + Z_u^0 h_u^0 + Z^1(u,V_u) h_u^1 + Z^2(u,V_u) h_u^2 \right) du \right\} \middle| \Im'_t \right]$$

Simple algebra gives us

$$V_t = E^{Q'}\left[\int_{]t,T]} e^{-\int_t^e (r_u + \tilde{h}_u) du} \left\{ dD_s + Z_s^0 h_s^0 ds + \left(\tilde{h}_s - h_s^0 + \mu(s,V_s) \right) V_s ds \right\} \middle| \Im'_t \right]$$

Integrating the linear terms gives us the desired results.

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